E. A. Stone
1915.

SECONDARY STRESSES

IN

BRIDGE TRUSSES

BY

C. R. GRIMM, C.E.

MEMBER OF THE AMERICAN SOCIETY OF CIVIL ENGINEERS,
MEMBER OF THE AMERICAN ASSOCIATION FOR
THE ADVANCEMENT OF SCIENCE, MEMBER
OF THE AMERICAN GEOGRAPHIC
SOCIETY

FIRST EDITION
FIRST THOUSAND

NEW YORK
JOHN WILEY & SONS
LONDON: CHAPMAN & HALL, LIMITED
1908

Stanhope Press
F. H. GILSON COMPANY
BOSTON, U.S.A.

PREFACE.

THIS book owes its existence to Dr. J. A. L. Waddell, Member of the American Society of Civil Engineers, who proposed to the writer the treatment of secondary stresses as a timely undertaking.

A comprehensive treatise on secondary stresses with numerous numerical examples involves an extraordinary amount of time and labor in its preparation, and considering the very limited amount of time at the writer's disposal for such an attempt, he thought it best to confine himself to a narrower field, as otherwise the publication would surely have been unduly delayed. It is owing to these circumstances that we discuss principally the most important secondary stresses, namely, those which are due to riveted joints in trusses.

We give in substance four principal methods of calculation of secondary stresses, which the reader may study separately and apply. The principle of least work leads also to the desired results, but it is far removed from shortening the labor. The nature of the subject forbids a very quick determination of the stresses, and definite rules or formulas cannot be given.

The problem of secondary stresses has been put in the proper light and the theories worked out entirely by German authors who have furnished also the greatest and best part of the contributions to the literature of this subject.

In preparing these notes the writer consulted a great many papers and books most of which are noted in Chapter XI. He has endeavored to be clear and precise in his statements, but if he has failed in this he begs the reader to be indulgent with his shortcomings. Should he succeed in interesting his readers in this new field for American bridge literature, and in rendering some service to his colleagues, he would consider himself richly repaid for his labors.

C. R. G.

GREATER NEW YORK, 1908.

CONTENTS.

CHAPTER I.

	Page
GENERAL AND HISTORICAL NOTES	1–5
Distinction between Primary, Principal, Additional and Secondary Stress	1
Deformation of Trusses	2
The Most Fruitful Sources of Secondary Stresses	2
Displacement of Stresses	2
Secondary Stresses as Higher Functions of the Exterior Loads	3
Origin of the Expression "Secondary Stress"	3
Solution of the Problem by Manderla	3
Difficulties of the Problem	4
Further Contributions to the Subject by Engesser, Winkler, Landsberg, Müller-Breslau, Ritter and Mohr	4
Cases of Examination of Secondary Stresses	5

CHAPTER II.

NATURE OF THE PROBLEM AND MEANS FOR ITS SOLUTION	6–19
Deflection of a Pin-Connected and a Riveted Truss	6
Manderla's Supposition	6
Deformation of Riveted Members, Resembling Usually the Letter S	7
The Angle of Deflection	7
The Conditions of Equilibrium for a Deformed Bar	7
Solution Effected by Means of the Equation of the Elastic Line	8
Simultaneous Performance of the Calculation for the Entire Truss Load	8
Simplified Assumptions — the Method of Influence Lines	9
Disturbing Influences	9
Determination of the Angle-Alterations $\Delta\alpha$	10
Relations between the Angle of Deflection and the End Moments	12
Determination of the Angle Included Between Successive Positions of the Axis of a Bar, Belonging to a Truss with Frictionless Pins, which is Acted Upon by Exterior Forces	16
Work Done by a Couple	17
The Elastic Line of a Straight Beam Represented as an Equilibrium Curve after Mohr	17

CHAPTER III.

	PAGE
MANDERLA'S METHOD	20–34
Assumptions Made for the Solution of the Problem of Secondary Stresses Caused by Riveted Joints	20
Other Elements of Influence than Riveted Joints on Secondary Stresses	20
Manderla's Course of Investigation	20
Differences between Compression- and Tension-Members	21
The Deflection Angles	24
Compression-Members	25
Tension-Members	27
Determination of the Deflection-Angles from the Conditions of Equilibrium	30
Secondary Stresses Found by Trial-Computations	33

CHAPTER IV.

MÜLLER-BRESLAU'S METHOD	35–43
Derivation of the Fundamental Equations	35
Assumed Deformation of Triangles	35
Values of the Deflection Angles	36
Determination of the Quantities U for a Truss	38
Method of Influence Lines	40

CHAPTER V.

RITTER'S METHOD	44–51
Notation of Quantities	44
Fundamental Equations	46
Graphic Solution	46
More Exact Method	49

CHAPTER VI.

MOHR'S METHOD	52–57
Determination of the Unknown Quantities	52
Determination of the Angles ψ, Pratt Truss as Example	53
The Bending Moments Expressed as Functions of the Angles ϕ and ψ	54
Resulting Equations and their Solution	57

CHAPTER VII.

PAGE

METHOD OF LEAST WORK 58–67
A Riveted Triangle as an Example 58
The Unknowns of the Problem 59
Influence of the Sectional Areas on the Stresses 59
Application of the Principle of Least Work 60
Determination of the Bending Moments and the Secondary Stresses 63
Displacement of the Stresses 64
Test of Accuracy by a Check-Calculation 64

CHAPTER VIII.

OTHER CAUSES OF SECONDARY STRESSES THAN RIVETED JOINTS IN MAIN TRUSSES 68–84
Eccentricities 68
Loads Between Panel-Points in the Plane of the Truss 69
Effects of Dead Load 69
Loads Between and at the Panel-Points of a Member Supposed to Turn Freely Around a Pin 69
Bottom-Chord Eyebar as an Example 70
Changes in Temperature 73
Misfits 74
Brackets on Posts 75
Unsymmetrical Connections 75
Curved Members 75
Pin-Joints 75
Greatest Diameter of a Pin, if an Eyebar Shall Turn Freely . . . 76
Amount of Secondary Stress in an Eyebar Due to Frictional Resistance 77
Friction at Supports 78
Cross-frames 78
Amounts of Secondary Stresses in the Suspenders of a Two-Track Railway Through Bridge 79
Amounts of Secondary Stresses in the Posts of a Four-Track Railway Through Bridge 79
Yielding of Foundations and Settlements of Masonry 80
Trusses Affected by a Displacement of Their Supports 80
Continuous Truss or Girder Over Three Supports 81
Two-Hinged Arch 81
Horizontal Thrust of a Two-Hinged Arch Caused by Vertical Loading, Changes in Temperature and Horizontal Yielding of the Supports 83
Bridge Across the Emperor William Canal at Grünenthal, Germany, as an example 83

CHAPTER IX.

PAGE

IMPACT 85–91
The Propagation of Stresses in Elastic Bodies and the Laws of Sound 86
Ritter, Mach, Radinger and Steiner on the Propagation of Impulses and Vibrations 86
Radinger on Bridge-Stresses by Fast Running Trains 87
Zimmermann's Rigid Solution of the Problem of Impact in the Simplest Case 87
Zimmermann's Impact Formula 90

CHAPTER X.

EXAMPLES AND CONCLUDING REMARKS 92–137
Amounts of Secondary Stresses in Three Warren Trusses without Verticals 94–99
Amounts of Secondary Stresses in Two Warren Trusses with Verticals 100–104
Discussion of Secondary Stresses 105–106
Amounts of Secondary Stresses in Three Pratt Trusses. 107–115
Amounts of Secondary Stresses in Two Double Intersection Warren Trusses without Verticals 116–119
Amounts of Secondary Stresses in One Double Intersection Warren Truss with Verticals 120–122
Discussion of Secondary Stresses continued 123
Amounts of Secondary Stresses in a Parabola Truss 124–125
Amounts of Secondary Stresses in a Continuous Truss 126–127
Discussion of Secondary Stresses continued 128
Points to be Observed for the Reduction of Secondary Stresses in Trusses Due to Static Loads 128–130
Secondary Stresses Due to Riveted Connections Between Floor-beams and Main Trusses 130–131
Remarks on Floorbeams 132
Points to be Observed for the Reduction of Impact and Vibrations 132–133
Bridge Collisions 133–135
Remarks on Calculations and their Use 135–137

CHAPTER XI.

LITERATURE 138–140
Concerning Secondary Stresses 138–140
Concerning Impact and Vibrations 140

SECONDARY STRESSES IN BRIDGE TRUSSES.

CHAPTER I.

GENERAL AND HISTORICAL NOTES.

Before entering upon a discussion of secondary stresses, a subject which has been very obscure up to about twenty-five years ago, it is proper to show to the reader the distinction the Germans make between different stresses, so that he can see at the outset the meaning they attribute to the expression, "Secondary Stress."

A bridge may be exposed to the influence of a number of causes, dead load, live load, wind pressure, centrifugal and braking forces, changes of temperature, yielding of masonry (as, for instance, in statically indeterminate arches, continuous beams and trusses), impact, etc. Any one of these causes produces primary stresses, which are separated into two classes. Such that are due entirely to dead load and live load are called by German writers *Hauptspannungen* (principal stresses), while those due to any other cause are called *Zusatzspannungen* (additional stresses). The resultant of the primary stresses passes through the center of gravity of the section and acts along the axis of the member, producing either an elongation or a shortening. The remainder of the stresses are bending, shearing and torsional stresses; they bend, displace and twist, and are comprised under the expression, secondary stresses (German — *Nebenspannungen*, *Sekundärspannungen;* French — *efforts secondaires*).

In other words, in a truss with frictionless pins, representing an ideal truss, the axes of the truss members remain straight during deformation, while in a riveted truss these axes are subjected to deformations accompanied by secondary stresses.

In the computations of bridge stresses we always proceed under the supposition that the joints are provided with frictionless pins; a supposition, which is either not at all realized in the structure, or at best in an imperfect manner, so that each truss member is bound to be deformed, that is to say, each member during the action of the load is subjected to bending moments in the plane of the truss, and its axis cannot remain straight. Not only the members composing a riveted truss, but all those that go into the riveted cross frames and the bracings are subjected to secondary stresses. The riveted joints of the main trusses in particular, as also the stiff connections between floor system and trusses form the most fruitful source of secondary stresses. For this reason, we will give for the present some general remarks about the secondary stresses in riveted trusses and discuss later those stresses which arise from the riveting between main trusses and floorbeams, as also several other groups of secondary stresses.

The deflection of an ideal truss with frictionless pins, due to external forces, causes alterations of leverarms, which the computer of primary stresses completely ignores, and rightfully so, since these alterations have no practical significance whatever. The original leverarms are very great in comparison with the alterations that have taken place after deflection and, therefore, these changes are neglected. The lines of stresses of the different members around a panelpoint still pass through a common center. On the other hand, when we attempt the computation of secondary stresses in a riveted truss, we have to deal with small leverarms, because each member of the truss has been bent and the alterations of these leverarms cannot be ignored without some sufficient reason. The resultant stress in each bar does not act any longer parallel to the axis of the bar, on the contrary, it forms an angle with it, which makes the leverarm and the lines of stresses of the

different members around a panelpoint pass no longer through one and the same mathematical point, that is, through a panelpoint of the truss.

By taking into account the influence of the deformations on the alterations of leverarms, the secondary stresses appear as higher functions of the exterior forces, while in neglecting these influences they appear as linear functions of the exterior forces, and one of the difficulties is removed.

It is natural that the deformation of a compression member plays a greater rôle than that of a tension member, but if ample provision against buckling is made in the design of compression members, the neglect of the deformations in the calculations is justified.

The expression, *Sekundärspannung* (secondary stress), originated with Professor Asimont of the polytechnic school in Munich, Bavaria. In a paper, "Hauptspannung und Sekundärspannung" (primary and secondary stress), which was published in 1880 in *Zeitschrift für Baukunde*, Asimont discusses the effects of eccentric loading on a column, and here he made the distinction between primary or direct stress and that due to a couple, producing a bending stress, which he called *Sekundärspannung*. But long before Asimont coined that expression, engineers considered secondary stresses in their designs.

The subject of secondary stresses being one of importance, the polytechnic school in Munich, in the year 1877, offered a prize on the solution of the problem of how to calculate these stresses in riveted trusses. In formulating the problem, Asimont suggested that, owing to the fact that the lines of the resulting stresses no longer pass through the centers of the paneljoints, its solution might be effected by the employment of Euler's equation of the elastic line. The prize was awarded to Manderla, in 1879, whose excellent solution is found in a highly scientific and mathematical paper, published in 1880, in *Allgemeine Bauzeitung*, under the title, "Die Berechnung der Sekundärspannungen, welche im einfachen Fachwerke infolge starrer Knotenverbindungen enstehen" (The calculation of secondary stresses which occur in simple trusses as a

consequence of rigid joints). The nature of the problem of secondary stresses in riveted trusses is such that it offered great obstacles to a solution; in fact, this problem is one of the most difficult in technical mechanics, and although Manderla's solution is a very great step forward, the problem in all its aspects has not yet been completely solved.

Before the appearance of Manderla's solution, Engesser had published an approximate method. In the year 1881, the late Professor Winkler gave a lecture on secondary stresses before an organization of engineers and architects in Berlin, in which he said that for some years past he had paid attention to the subject. This lecture is published in *Deutsche Bauzeitung* in 1881, under the title, "Die Sekundärspannungen in Eisenkonstruktionen" (Secondary stresses in iron constructions).* In 1885 Professor Landsberg contributed a graphical solution under the assumption that the chords alone are riveted; and in 1886 Professor Müller-Breslau made an analytical contribution. Professor Ritter, in 1890, gave a graphical solution, and in the years 1892 and 1893 Professor Engesser published a book on secondary and additional stresses, the latter expression to be taken in the sense already explained. In this book a systematic representation of the whole subject is given; the treatment is analytical throughout. Professor Mohr contributed in 1892 an analytical method in "*Der Civil Ingenieur*," which can also be found in his very valuable book, "Abhandlungen aud dem Gebiete der technischen Mechanik," published in 1906, in which he treats in a masterly manner subjects appertaining to technical mechanics.

The fundamental truths and the methods of calculations that came to light in studying the problem that is here under consideration, we owe to German scientists. The results they obtained were soon made use of by German bridge constructors, as is noticeable in their designs, but this is a point to which we will return at another place.

* Very extensive investigations can be found in Winkler's "Theorie der Brücken," II Teil.

The calculations of secondary stresses are very extensive and require therefore much time; but under simplified assumptions, and with the requirement to find these stresses for only one given load system, the calculations can be performed with comparative speed. It is hardly in the nature of the problem to give empirical rules and formulas. Although in common cases there is no necessity for such calculations, yet in particular cases secondary stresses should be investigated; for instance, in cases where we can expect them to be of great magnitude, or where a bridge has to carry much greater loads than those for which it has been designed, which is true of old bridges.*

The margin of safety provided for in our specifications must cover the secondary stresses without impairing the safety of the structure.

The nature of a truss, its details, as well as the dimensions of its individual members, are of great importance for the reduction of secondary stresses, but in order to obtain the best results even attention must be paid to the manufacture, erection and maintenance of bridges.

* W. J. Watson, Concerning the investigation of Overloaded Bridges, Proceedings Am. Soc. of C. E., April, 1906.

CHAPTER II.

NATURE OF THE PROBLEM AND MEANS FOR ITS SOLUTION.

IN the following discussion it is assumed that all exterior forces, to whose influence a truss is exposed, are acting in the plane of the truss. The deflection of a truss with supposed frictionless pins is, strictly speaking, not the same as that of a riveted truss, all other conditions being the same for both trusses. While in the former case the problem is geometrical, since the bars can turn freely around the pins and remain straight, in a riveted truss the axes of the bars become deformed under the action of the load, a fact which must be considered, together with the moments of inertia of the sections, in determining the deflection. But the difference in the deflection between these two cases is so small that its consideration has no practical value.

Manderla's solution is based on the supposition that the positions of the panelpoints in a riveted truss under the action of outer forces are the same as if the truss were provided with frictionless pins.

In Fig. 1, representing a fragment of a truss, the angle formed by the two bars 01 and 02 before any deformation takes place is $\measuredangle\ 102 = \alpha$. Under the supposition of frictionless pins this angle α will be changed as soon as the truss is subjected to the influence of exterior forces. Let this change be $\Delta\alpha$, so that the angle included between the two bars 01 and 02 after deformation equals $\alpha + \Delta\alpha$.

If we now conceive the truss to have joints riveted in such a manner as to keep the ends of the bars absolutely fixed, then under the influence of the outer forces the originally straight bars will be deformed and the angle included between the two end

tangents oT_1 and oT_2, drawn to the elastic lines, which are the deformed axes of the bars, will remain unchanged during deformation, that is to say, the angle $T_1oT_2 = \alpha$ or = the original angle before any deformation took place.

Each bar will be subjected to bending moments, and its deformation generally, but not always, resembles the letter S. The bars

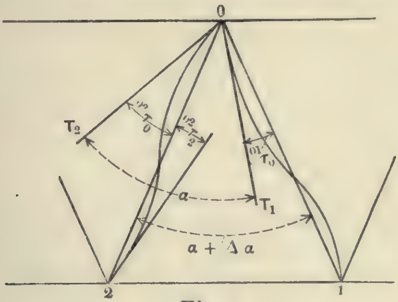


Fig. 1.

must be considered fixed at the ends and under the influence of an axial load, while the effect of the bending moments can be conceived as consisting in the reduction of the changed angle $\alpha + \Delta\alpha$ to its original magnitude α.

The angle τ, included between a chord of a deformed bar and its corresponding end tangent, as, for instance, oI and oT, we will call the angle of deflection.

We will now consider a single bar all by itself, for instance, bar o1 (see Fig. 2), by passing sections close to the panelpoints and applying the resultant P_0 of all the interior forces, which intersects the chord of the deformed bar under the angle ω. This resultant is resolved into the force S_0, acting along the axis of the deformed bar, the transverse force Q_0 and the moment M_0.

The equilibrium requires that

$$S_0 = S_1,\ Q_0 = Q_1,\ \frac{M_0 + M_1}{l} - Q_1 = 0.$$

The magnitude of the transverse force $Q = \dfrac{M_0 + M_1}{l}$ is princi-

pally dependent on the signs of the two moments; the greater values of Q correspond to equal signs of M and consequently to a double curvature of the bar, while opposite signs of the moments mean smaller values of Q and a single curvature. The transverse force Q is constant for the entire length of the bar, as its ends are the only points of application of the outer forces, and the bending moment for any point between the ends of the bar is variable under the influence of Q and S.

Manderla's excellent original solution considers all of the forces represented in Fig. 2. He proceeds from the equation of the elastic line, employs for the integrations hyperbolic functions, and after a relation between the bending moments and the angle of deflection τ is established, expressing the latter in terms of α, he obtains the desired result by trial.

Other scientists, who have occupied themselves with a solution of this problem, proceed from the assumption that the influence of the deformation on the leverarm y is a negligible quantity, or in other words, for well designed compression members —tension members are not so important in this respect — the leverarm y, and consequently also the moment Sy, is so small that it does not need to be considered. The transverse forces Q, too, are in most cases small enough to be neglected. These suppositions lead to simplified calculations.

In Manderla's solution the secondary stresses appear as higher

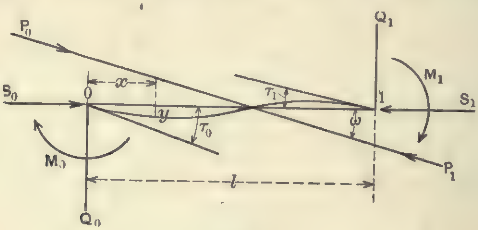


Fig 2.

functions of the exterior forces, which means that the calculations must be performed simultaneously for the entire load system.

If, for instance, the secondary stresses in a railroad truss have been determined for a certain position of a train, we have no means of knowing what the magnitude of these secondary stresses amount to as soon as the train is moved to some other position. The method of influence lines does not hold true in this case, while under the simplified assumption, that is, if the effect of the moment Sy on the final result is neglected, the secondary stresses appear as linear functions of the exterior forces and the investigator may employ the method of influence lines if he so desires. The fact that secondary stresses do not increase or decrease in direct proportion to the exterior loads makes the so-called "factor of safety"* appear as the "factor of uncertainty," and uncertainties must be covered by the margin of safety in our specifications.

The assumptions which we have made are never strictly realized in a structure, but this is, of course, equally true for the calculations of other bridge stresses. It is within the range of possibilities that the deformation of some truss members will take place outside of the plane of the truss and they may even be subjected to the influence of torsional moments. It must also be borne in mind that the desire of the bridge engineer to save some weight often results in sizes of gusset plates so small that the state of fixity of the bars is complied with in a very imperfect manner. A variation in the value of the modulus of elasticity and possible erection stresses play also a rôle in making the results somewhat uncertain.

In the first place we will deduce some fundamental conclusions from technical mechanics, which are indispensable for a clear understanding of the different methods of calculations of secondary stresses.

* (1) "Factors of safety," *Eng. News*, Sept. 6, 1906.

(2) "The Investigation of Old Bridges a Phase of Maintenance Engineering," *Eng. News*, Sept. 13, 1906.

1. Determination of the Angle Alterations $\Delta\alpha$.

An important problem in the theory of trusses, forming the foundation of Manderla's solution, consists in the determination of the alterations $\Delta\alpha$ of the angles α between any two adjacent pin-connected bars of a truss when the latter is acted upon by outer forces. $\Delta\alpha$ will be expressed in terms of the primary stresses, which are known quantities.

The deformation of the angle α, in the triangle ABC of Fig. 3, is due to the elastic changes of all three sides, and this deformation is obtained by determining the influence of each bar on the deformation successively, whereupon the results are summed up. This is done in assuming that each bar in turn is elastic, the other two being non-elastic. If the side AC is alone elastic, experiencing a contraction to the amount of ΔL_3, it will be forced to revolve around the apex A, while the side BC revolves around B, the side AB being supposed to be held fast.

From the figure we have

$$\Delta\alpha_{13} = \alpha_1 - \beta_1,$$

where the index 1 refers to the angle and the index 3 to the side.

Further:

$$\alpha_1 = 180^\circ - \alpha_2 - \alpha_3$$

and

$$\beta_1 = 180^\circ - \alpha_2 - \alpha_3 - \varepsilon + \gamma$$

consequently:

$$\Delta\alpha_{13} = -\gamma + \varepsilon.$$

The angle γ is found from the equation $BC\gamma \times \cos\alpha_1 = a$ or $\gamma = \dfrac{a}{BC\cos\alpha_1} = \dfrac{a}{b} = \dfrac{\Delta L_3}{h}$ on account of the similarity of the triangles BCF and CDE. We have also $\dfrac{a}{b} \times \dfrac{b}{L_3} = \dfrac{b\Delta L_3}{L_3 h}$, and since the angle $\varepsilon = \dfrac{a}{L_3}$, we find by substitution: $\varepsilon = \dfrac{b\Delta L_3}{L_3 h}$.

With these values for γ and ϵ the deformation becomes

$$\Delta\alpha_{13} = -\frac{\Delta L_3}{h} + \frac{b\Delta L_3}{L_3 h}.$$

If S_3 denotes the total stress in bar AC and A_3 its cross-section, then $\frac{S_3}{A_3} = s_3$, which is the stress per unit of area. Calling E the modulus of elasticity, the deformation of the angle α_1, due

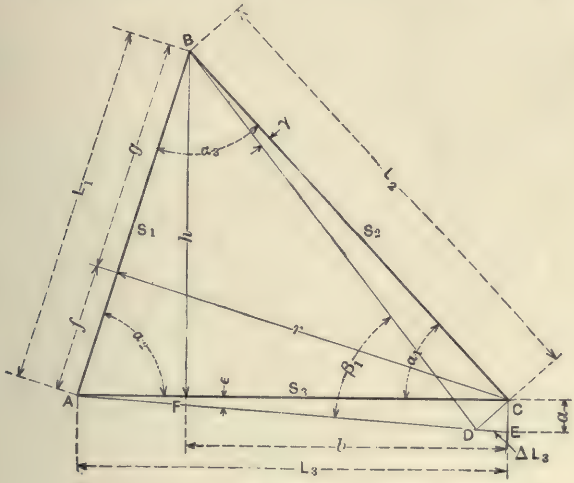


Fig. 3.

only to the alteration in the length of the bar AC, is expressed by

$$\Delta\alpha_{13} = -\frac{S_3}{EA_3} \times \frac{L_3 - b}{h} = -\frac{S_3}{EA_3}\cot\alpha_2 = -\frac{s_3}{E}\cot\alpha_2.$$

If the bar BC alone is elastic, we find in a similar manner,

$$\Delta\alpha_{12} = -\frac{s_2}{E}\cot\alpha_3,$$

and under the supposition that AB alone is subjected to a change in its length, we have:

$$\Delta\alpha_{11} = \frac{\Delta L_1}{r} = \frac{S_1 L_1}{EA_1 r} = \frac{S_1}{EA_1}\left\{\frac{f}{r} + \frac{g}{r}\right\} = \frac{s_1}{E}\left\{\cot\alpha_2 + \cot\alpha_3\right\}.$$

The total deformation of angle α_1 is now found by summing up the three results, just found, so that we can write:

$$\Delta\alpha_1 = \Delta\alpha_{11} + \Delta\alpha_{12} + \Delta\alpha_{13}.$$

In substituting the values in this equation and treating the deformations of the angles α_2 and α_3 in like manner, we finally arrive at the three equations:

$$\left.\begin{aligned}
\Delta\alpha_1 &= \left\{\frac{s_1 - s_2}{E}\right\}\cot\alpha_3 + \left\{\frac{s_1 - s_3}{E}\right\}\cot\alpha_2 \\
\Delta\alpha_2 &= \left\{\frac{s_2 - s_3}{E}\right\}\cot\alpha_1 + \left\{\frac{s_2 - s_1}{E}\right\}\cot\alpha_3 \\
\Delta\alpha_3 &= \left\{\frac{s_3 - s_1}{E}\right\}\cot\alpha_2 + \left\{\frac{s_3 - s_2}{E}\right\}\cot\alpha_1
\end{aligned}\right\} \qquad (1)$$

If we attribute to a tensile stress the positive and to a compressive stress the negative sign, we can obtain, for instance, the positive maximum deformation of angle α_1 by supposing s_1 a tension and s_2 and s_3 a compression, while the negative maximum deformation of the same angle results from a negative s_1 and a positive s_2 and s_3.

2. Relations Between the Angle of Deflection and the End Moments.

The Italian engineer Alberto Castigliano in his book, "Théorie de l'équilibre des Systemes élastiques," demonstrated and introduced the principle of the derivative of work and the principle of least work.* The first quoted principle means, in our case, that if we express the work of deformation of a bar as a function of the outer forces, its first derivative with respect to a moment equals the angle of revolution of the bar. If the first derivative is taken with respect to a force, we obtain the displacement in the direction of the force of its point of application.

In Fig. 4, let AB represent a beam fixed at B and under the influence at its end A of a vertical force Q and a moment M_0.

* W. Cain: Determination of the Stresses in Elastic Systems by the Method of Least Work. Transactions Am. Soc. C. E., Vol. XXIV, 1891.

The angle τ included between a horizontal line and the end tangent drawn to the elastic line of the beam is the angle sought,

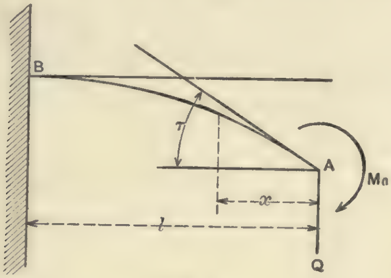


Fig. 4.

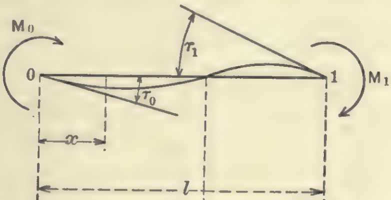
measured by the length of a circular arc (of unit radius) and expressed by the equation:

$$\tau = \frac{\int_0 \frac{M^2}{2\,EI}\,dx}{\partial M_0} = \int_0 \frac{M}{EI}\frac{\partial M}{\partial M_0}\,dx.$$

M = Moment around any point of the axis of the beam,
E = Modulus of elasticity,
I = Moment of inertia.

We will now consider a bar of double curvature, Fig. 5, but with

Fig. 5.



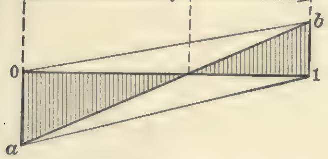


Fig. 6.

the express understanding that the end moments only will be considered in determining the deflection angles τ.

In Fig. 6, the moments are represented graphically, $M_0 = oa$ and $M_1 = 1\, b$, and the moment area appears as the difference of two triangles, so that we can write:

$$\text{Area } o1ba = \Delta oab - \Delta o1b.$$

The moment in reference to any point of the axis of the bar or

$$M = -\{M_0 + M_1\}\frac{x}{l} + M_0$$

and

$$\frac{\partial M}{\partial M_0} = -\frac{x}{l} + 1.$$

Applying now the principle of the derivative of work for the determination of the angle τ as expressed in the above equation, we can write:

$$\tau_0 = \int_0^l \frac{M}{EI}\frac{\partial M}{\partial M_0}dx =$$

$$\frac{1}{EI}\int_0^l \left[\left\{-(M_0 + M_1)\frac{x}{l} + M_0\right\}\left\{-\frac{x}{l} + 1\right\}\right]dx$$

which gives:

$$\tau_0 = \frac{1}{EI}\left\{M_0\frac{l}{3} + M_1\frac{l}{3} - M_0\frac{l}{2} - M_0\frac{l}{2} - M_1\frac{l}{2} + M_0 l\right\}$$

or

$$\left.\begin{aligned}\tau_0 &= \frac{l}{6\,EI}\{2\,M_0 - M_1\}\\ \tau_1 &= \frac{l}{6\,EI}\{2\,M_1 - M_0\}\end{aligned}\right\} \qquad (2)$$

τ_1 is found by exchanging M_0 for M_1.

The deformation shows a single curvature and with no point of contraflexure when a bar is acted upon by two end moments of opposite sign, which case will hereafter be exclusively considered, Figs. 7 and 8.

In regard to Fig. 8, $M_0 = oa$ and $M_1 = 1\,b$, and the moment area appears as a trapezoid. The moment at any point of the axis of the bar is equal to

$$M = \{M_1 - M_0\}\frac{x}{l} + M_0$$

and
$$\frac{\partial M}{\partial M_0} = -\frac{x}{l} + 1,$$

Fig. 7

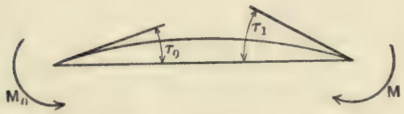
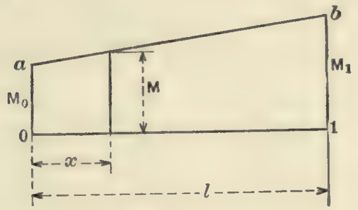


Fig. 8



consequently we have:

$$\tau_0 = \int_0^l \frac{M}{EI}\frac{\partial M}{\partial M_0}\,dx =$$

$$\frac{1}{EI}\int_0^l \left[\left\{(M_1 - M_0)\frac{x}{l} + M_0\right\}\left\{-\frac{x}{l} + 1\right\}\right]dx$$

or
$$\tau_0 = \frac{l}{6\,EI}\{M_1 + 2\,M_0\}$$

and also $\quad$ (3)

$$\tau_1 = \frac{l}{6\,EI}\{M_0 + 2\,M_1\}$$

3. Determination of the Angle Included Between Two Successive Positions of the Axis of a Bar, Belonging to a Truss with Frictionless Pins, which is Acted Upon by Exterior Forces.

We assume OA and OA_1 are the directions of the axis of the bar "before" and "after" deformation, so that the angle ψ included between OA and OA_1 is the angle sought; see Fig. 9. The equation

$$\sum P\delta = \sum S\Delta l$$

states that the work done by the exterior forces equals the work done by the interior forces in case of equilibrium. In this equation P designates an outer force and S its corresponding stress, while Δl is an alteration of the length of a bar and δ a displacement in the direction of the force P of its point of application.

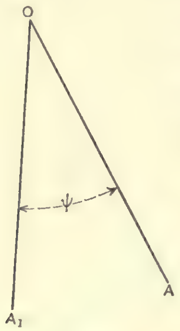


Fig. 9.

The work done by the reactive forces is not considered, as we assume that the points of support are immovable in the direction of these forces. The above equation holds true for any possible displacements δ and alterations Δl and for any values of P which are independent of the δ and Δl.

If we wish to calculate any particular δ_r in the direction of P_r, which is caused by a given load system, it is only necessary to let all of the forces P vanish, except P_r, which is put equal to unity. In doing this the new equation results:

$$1 \times \delta_r = \sum s\Delta l,$$

in which the stresses s are caused by the force unity and the Δl are caused by the given exterior forces, δ_r corresponding to the Δl. The force unity may represent a single force or a moment, in which latter case the angle of revolution ψ, measured by the length of the circular arc, must be substituted for the displacement δ_r and our equation becomes:

$$1 \times \psi = \sum s\Delta l. \tag{4}$$

We will now show that the expression $M \times \psi$ represents work as it should do.

In Fig. 10, OA represents a bar, having its center of revolution at O and acted upon by a moment $M = Q \times a$. The angle ψ is

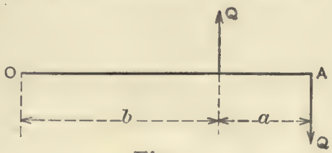


Fig. 10.

measured downward from OA in agreement with the direction of revolution of the moment $Q \times a$, which is the same as that of the hands of a clock.

From the figure we have:

$$\begin{aligned} \text{Work} &= Q \times \{a + b\}\, \psi - Q \times b\psi \\ &= Q \times a\psi = M \times \psi. \end{aligned}$$

4. The Elastic Line of a Straight Beam Represented as an Equilibrium Curve after Mohr.*

This problem was first solved by Mohr nearly forty years ago

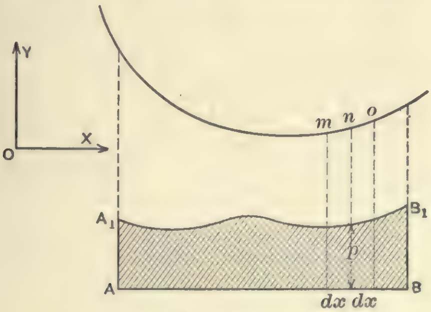


Fig. 11.

by comparing the equation of the elastic line with that of an equilibrium curve.

* Mohr: "Abhandlungen aus dem Gebiete der technischen Mechanik," 1906.

In Fig. 11, the beam is subjected to a vertical load, which is continuous but non-uniform. The hatched area is the load area of the beam and p designates the variable load per unit of length of the horizontal projection of the curve.

Figure 12 is a fragment of the force polygon and Fig. 13

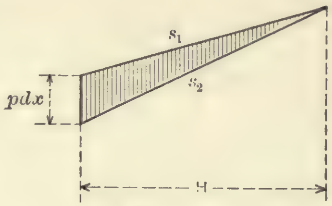


Fig. 12.

shows the infinitely small sides *mn* and *no* of the equilibrium polygon (curve) with horizontal and vertical projections in an

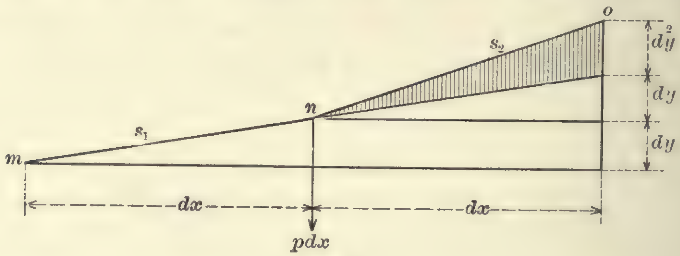


Fig. 13.

exaggerated scale, $p\,dx$ being the load at point n. As the hatched triangles are similar in Figs. 12 and 13, we can write:

$$\frac{d^2y}{dx} = \frac{p\,dx}{H}.$$

Dividing by dx we have the differential equation of the equilibrium curve for vertical forces, that is,

$$\frac{d^2y}{dx^2} = \frac{p}{H}.$$

The differential equation of the elastic line

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

is of the same form, where M = any moment, E = modulus of elasticity and I = moment of inertia. Consequently, if we put

$$M = p \text{ and } EI = H,$$

or $$\frac{M}{I} = p \text{ and } E = H,$$

or $$\frac{M}{EI} = p \text{ and } 1 = H,$$

we see that the elastic line can be conceived as an equilibrium curve.

CHAPTER III.

MANDERLA'S METHOD.

THE problem of secondary stresses is solved under the assumption that the deformations of a riveted truss, caused by the exterior loading, are accomplished in the plane of the truss and that no torsion exists. It is further assumed that every load is applied at the panelpoints of the truss, which is composed of members having uniform sections. This last assumption is not quite true, since the truss members are connected together by means of gusset plates, which make a sudden change in the sectional areas at the ends of the members.

There are other elements which are of influence on the stresses, but which can be examined separately. One-sided connections hardly need any consideration, as they are condemned by our specifications; but the effect of details not centrally designed, as also the effect of loads, applied at any point between the panelpoints, as dead and live load, wind pressure and centrifugal force, and the influence of a change in temperature on the secondary stresses, may be calculated. Of considerable importance is the effect of the deformations of floorbeams riveted to the trusses on the stresses in truss members, but this is a problem whose analytical discussion is entirely outside the range of our considerations.

The progress of the investigation is as follows: Proceeding from the supposition that the positions of the panelpoints of a riveted truss under the influence of loads are the same as if the truss had frictionless pins, the alterations of all the angles in the triangles, composing the truss, are calculated according to equations (1) in Chapter II, for that system of loads for which we intend to determine the secondary stresses. But, as the ends of our truss members are supposed to be rigidly fixed, any alterations

in the angles of the triangles are impossible, in consequence of which the truss members become deformed under the action of the exterior loads and are subjected to bending moments, which must reduce the supposed changed angles to their original magnitudes. The end bending moments are now introduced into the calculations as the unknowns, and relations are determined between them and the angles of deflection, which latter angles are included between the chords of the deformed bars and the end tangents drawn to the elastic lines of the bars. Hereupon the angles of deflection are expressed in terms of the alterations of the angles in the triangles calculated by means of equations (1) for the desired load system under the assumption of frictionless pins. After the unknown bending moments have been determined, it is easy to find the secondary stresses according to known formulas. Manderla finds the desired stresses not directly, but by a few trial computations, which are easily performed.

Before we proceed with the analysis of the stresses, it will be necessary to give some remarks on the action of the forces to which the truss members are subjected, and to point out the differences which exist between the compression and tension members, as shown in Figs. 14, 15, 16, 17, 18, and 19.

If we pass a cut close to a panelpoint and apply the inner forces to establish the equilibrium, then these forces can be represented by their resultant P, which in turn can be resolved into an axial force S, a transverse force Q, and a moment M. We refer the reader here, to what has been said on this subject in Chapter II. The two moments for each bar either act in the same or in the opposite sense. If in the same sense, the deformed bar is either on one side of its chord or partly on one and partly on the other side of the chord, and in both cases that place where the line of the resultant P intersects with the elastic line is a point of inflection. Should the two moments act in opposite directions, then the deformed bar is on one side only of its chord, and it has no point of inflection. The resultant P intersects with the prolongation of the chord of the bar ab.

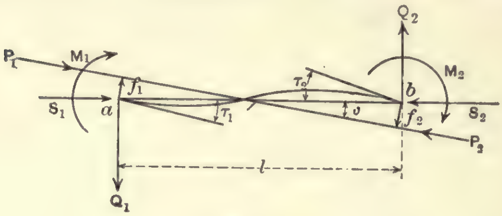


Fig. 14.

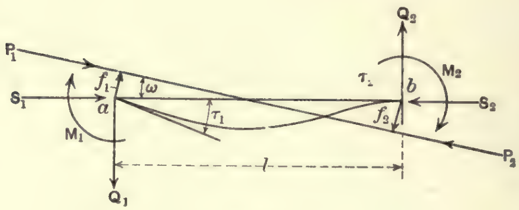


Fig. 15.

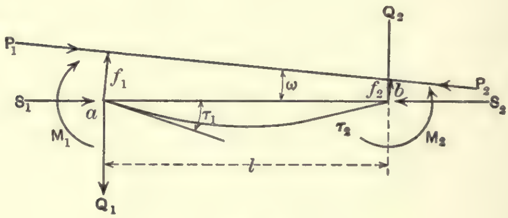
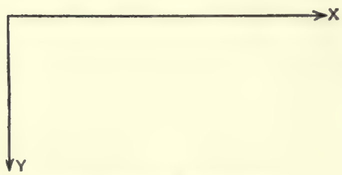


Fig. 16.



COMPRESSION MEMBERS.

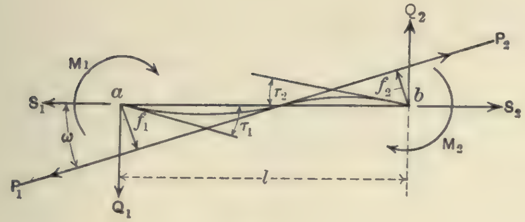


Fig. 17.

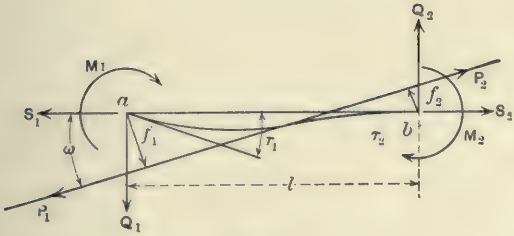


Fig. 18.

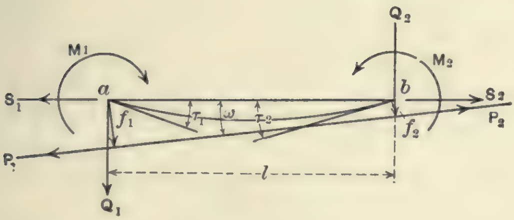


Fig. 19.

TENSION MEMBERS.

For tension members, the convex side of the elastic line is turned towards the line of the resultant P, and consequently no moment between the end points a and b of the bar can be the maximum moment.

For compression members, the concave side of the elastic line

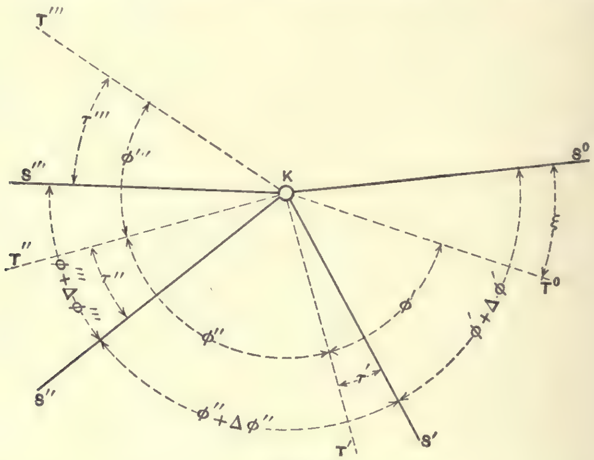


Fig. 20.

is turned towards the line of the resultant P, and a maximum moment between the ends a and b of the bar is possible.

We will now turn our attention to the deflection angles.

Figure 20 represents a panelpoint K where four bars intersect. The straight bars are shown in full lines, and any angle included between two adjacent bars KS, which had the value ϕ before deformation, becomes $\phi + \Delta\phi$ after deformation. The assumption is that during the action of the actual exterior loads upon the truss each bar is free to turn around a frictionless pin. But in reality the bar ends are fixed by riveting, consequently they must be deformed while the truss deflects, and the angles formed by two adjacent end tangents KT of the deformed bars, shown in dashed lines, must remain constant during deflection, or, in other words, these angles are the angles ϕ, which existed before the exterior loads began to act.

In Fig. 20, the angles of deflection τ are produced by clockwise revolution and are to be taken positive.

From the figure we have

$$\xi + \phi' = \phi' + \Delta\phi' + \tau',$$
$$\xi + \phi'' + \phi' = \phi'' + \Delta\phi'' + \phi' + \Delta\phi' + \tau'',$$
$$\xi + \phi''' + \phi'' + \phi' = \phi''' + \Delta\phi''' + \phi'' + \Delta\phi'' + \phi' + \Delta\phi' + \tau_{'''},$$

or

$$\tau' = \xi - \Delta\phi',$$
$$\tau'' = \xi - \{\Delta\phi' + \Delta\phi''\},$$
$$\tau''' = \xi - \{\Delta\phi' + \Delta\phi'' + \Delta\phi'''\},$$

and generally

$$\tau = \xi - \sum\Delta\phi. \tag{5}$$

This shows that if we knew only one deflection angle ξ, then from it and the $\Delta\phi$ all others around a panelpoint could be readily calculated.

1. Compression Members.

Our next step consists in seeking a relation between the moments M and the angles of deflection τ, and for this purpose we employ the equation of the elastic line, which we write, bearing in mind the directions of the axes of the coördinates as shown in Figs. 14 to 19,

$$\frac{d^2y}{dx^2} = -\frac{M_x}{EI}$$

where E = modulus of elasticity, I = moment of inertia and M_x = a moment at any point of the axis of the bar, that is,

$$M_x = + Sy - Q_1x + M_1.$$

Substituting and putting the three constants $\frac{S}{EI} = T^2$,

we have
$$\frac{d^2y}{dx^2} = -T^2y + \frac{Q_1x}{EI} - \frac{M_1}{EI},$$

and by integration

$$y = A \sin xT + B \cos xT + \frac{Q_1x}{T^2EI} - \frac{M_1}{T^2EI}. \tag{6}$$

The constant B is determined from the condition that y becomes o for $x = 0$, consequently

$$B = \frac{M_1}{S}.$$

The constant A is found by putting $y = 0$ for $x = l$, that is,

$$A = \frac{M_1}{S} \tan \frac{\rho}{2} - l \tan \omega \frac{1}{\sin \rho}.$$

In this last equation $\rho = lT = l\sqrt{\frac{S}{EI}}$, and $\tan \omega = \frac{Q_1}{S}$. If we put in equation (6) the values for the constants A and B, we get by differentiation,

$$\frac{dy}{dx} = \frac{M_1 T}{S}\left\{\tan \frac{\rho}{2} \cos xT - \sin xT\right\} + \tan \omega \left\{1 - \frac{\rho}{\sin \rho} \cos xT\right\}. \quad (7)$$

The equation (7) gives the angles of deflection

$$\frac{dy}{dx} = \tau_1 \text{ for } x = 0 \text{ and } \frac{dy}{dx} = \pm\, \tau_2 \text{ for } x = l.$$

These angles are:

$$\left.\begin{aligned} \tau_1 &= \frac{M_1}{S} T \tan \frac{\rho}{2} - \tan \omega \left\{\frac{\rho}{\sin \rho} - 1\right\} \\ \pm\, \tau_2 &= -\frac{M_1}{S} T \tan \frac{\rho}{2} - \tan \omega \left\{\frac{\rho}{\sin \rho} \cos \rho - 1\right\} \end{aligned}\right\} \quad (8)$$

If we now put for brevity's sake,

$$\frac{\rho}{2 - \rho \cot \frac{\rho}{2}} + \cot \frac{\rho}{2} = 2\, m_c \text{ and } \frac{\rho}{2 - \rho \cot \frac{\rho}{2}} - \cot \frac{\rho}{2} = 2\, n_c,$$

or

$$\frac{1}{2 - \rho \cot \frac{\rho}{2}} = \frac{m_c + n_c}{\rho},$$

we obtain by addition of the equations (8),

$$\tan \omega = \frac{m_c + n_c}{\rho} \{\tau_1 \pm \tau_2\}, \tag{9}$$

and by subtraction,

$$M_1 = \frac{S}{T} \{m_c \tau_1 \pm n_c \tau_2\}, \tag{10}$$

and by exchange of the angles τ,

$$M_2 = \frac{S}{T} \{\pm\, m_c \tau_2 + n_c \tau_1\}. \tag{11}$$

For practical calculations it is best to express m_c and n_c in series as follows:

$$\cot \rho = \frac{1}{\rho} - \frac{\rho}{3} - \frac{\rho^3}{45} \ldots\ldots$$

$$\cot \frac{\rho}{2} = \frac{2}{\rho} - \frac{\rho}{6} - \frac{\rho^3}{360} \ldots\ldots$$

$$2 - \rho \cot \frac{\rho}{2} = \frac{\rho^2}{6} \left\{ 1 - \frac{\rho^2}{60} - \frac{\rho^4}{2520} \ldots\ldots \right.$$

$$\frac{1}{2 - \rho \cot \frac{\rho}{2}} = \frac{6}{\rho^2} - \frac{1}{10} - \frac{1}{1400} \rho^2 \ldots\ldots$$

$$m_c = \frac{4}{\rho} - \frac{2}{15} \rho - \frac{11}{6300} \rho^3$$

$$n_c = \frac{2}{\rho} + \frac{1}{30} \rho + \frac{13}{12600} \rho^3 \ldots\ldots$$

2. Tension Members.

The moment M_x at any point of the axis of a tension member is

$$M_x = -\, Sy - Q_{1x} + M_1,$$

and consequently

$$\frac{d^2y}{dx^2} = T^2 y + \frac{Q_1 x}{EI} - \frac{M_1}{EI}.$$

By integrating we find

$$y = A\varepsilon^{-xT} + B\varepsilon^{xT} - \tan\omega \, . \, x + \frac{M_1}{S} \, . \qquad (12)$$

y becomes 0 for $x = 0$, or

$$A = \frac{M_1}{S} - B \text{ and } B = \frac{M_1}{S} - A.$$

As $y = 0$ for $x = l$, we get the values of the constants

$$A = + \frac{M_1}{S} \left\{ \frac{1 - \varepsilon^{\rho}}{\varepsilon^{\rho} - \varepsilon^{-\rho}} \right\} + l \tan\omega \frac{1}{\varepsilon^{\rho} - \varepsilon^{-\rho}},$$

$$B = - \frac{M_1}{S} \left\{ \frac{1 - \varepsilon^{-\rho}}{\varepsilon^{\rho} - \varepsilon^{-\rho}} \right\} + l \tan\omega \frac{1}{\varepsilon^{\rho} - \varepsilon^{-\rho}}.$$

Of course we have here again

$$T^2EI = S, \ \frac{Q_1}{S} = \tan\omega \text{ and } lT = \rho.$$

After substituting the values of the constants A and B in the equation (12), we obtain by differentiation the angle of deflection

$$\frac{dy}{dx} = \tau,$$

$$\frac{dy}{dx} = - \frac{M_1 T}{S} \left\{ \frac{1 - \varepsilon^{\rho}}{\varepsilon^{\rho} - \varepsilon^{-\rho}} \varepsilon^{-xT} + \frac{1 - \varepsilon^{-\rho}}{\varepsilon^{\rho} - \varepsilon^{-\rho}} \varepsilon^{xT} \right\}$$
$$+ \tan\omega \left\{ \frac{\varepsilon^{xT} + \varepsilon^{-xT}}{\varepsilon^{\rho} - \varepsilon^{-\rho}} \times \rho - 1 \right\} \qquad (13)$$

Further,

$$\frac{dy}{dx} = \tau_1 \text{ for } x = 0 \text{ and } \frac{dy}{dx} = \pm \tau_2 \text{ for } x = l,$$

or

$$\left.\begin{aligned} \tau_1 &= - \frac{M_1}{S} T \left\{ \frac{1 - \dfrac{\varepsilon^{\rho} + \varepsilon^{-\rho}}{2}}{\dfrac{\varepsilon^{\rho} - \varepsilon^{-\rho}}{2}} \right\} + \tan\omega \left\{ \frac{\rho}{\dfrac{\varepsilon^{\rho} - \varepsilon^{-\rho}}{2}} - 1 \right\} \\ \pm \tau_2 &= + \frac{M_1}{S} T \left\{ \frac{1 - \dfrac{\varepsilon^{\rho} + \varepsilon^{-\rho}}{2}}{\dfrac{\varepsilon^{\rho} - \varepsilon^{-\rho}}{2}} \right\} + \tan\omega \left\{ \frac{\dfrac{\varepsilon^{\rho} + \varepsilon^{-\rho}}{2}}{\dfrac{\varepsilon^{\rho} - \varepsilon^{-\rho}}{2}} \times \rho - 1 \right\} \end{aligned}\right\} \qquad (14)$$

It is suitable for our purpose to replace the exponential functions by hyperbolic functions, which latter we express in series.

The hyperbolic sine and the hyperbolic cosine are written:

$$\sin h\rho = \frac{\varepsilon^{\rho} - \varepsilon^{-\rho}}{2} \text{ and } \cos h\rho = \frac{\varepsilon^{\rho} + \varepsilon^{-\rho}}{2}.$$

We have further,

$$\cot h\frac{\rho}{2} = \frac{1 + \cos h\rho}{\sin h\rho} \text{ and } \tan h\frac{\rho}{2} = \frac{1 - \cos h\rho}{\sin h\rho},$$

and consequently the equations (14) are transformed into

$$\left.\begin{aligned} \tau_1 &= \frac{M_1}{S}\ T \tan h\frac{\rho}{2} + \tan\omega\left\{\frac{\rho}{\sin h\rho} - 1\right\} \\ \pm\tau_2 &= -\frac{M_1}{S}\ T \tan h\frac{\rho}{2} + \tan\omega\left\{\frac{\rho}{\sin h\rho} \times \cos h\rho - 1\right\} \end{aligned}\right\} \quad (15)$$

If we write

$$\frac{\rho}{\rho\cot h\frac{\rho}{2} - 2} + \cot h\frac{\rho}{2} = 2\, m_t,$$

$$\frac{\rho}{\rho\cot h\frac{\rho}{2} - 2} - \cot h\frac{\rho}{2} = 2\, n_t,$$

we find by addition,

$$\frac{1}{\rho\cot h\frac{\rho}{2} - 2} = \frac{m_t + n_t}{\rho}.$$

But

$$\frac{\rho}{\sin h\rho} - 1 + \frac{\rho}{\sin h\rho} \times \cos h\rho - 1 = \frac{\rho}{\sin h\rho}\{1 + \cos h\rho\} - 2$$

$$= \rho\cot h\frac{\rho}{2} - 2 = \frac{\rho}{m_t + n_t},$$

and in adding the equations (15) and using this last expression we obtain

$$\tan \omega = \frac{m_t + n_t}{\rho} \{\tau_1 \pm \tau_2\}, \tag{16}$$

and in subtracting them,

$$M_1 = \frac{S}{T} \{m_t \tau_1 \pm n_t \tau_2\}, \tag{17}$$

and by exchanging the angles τ,

$$M_2 = \frac{S}{T} \{\pm m_t \tau_2 + n_t \tau_1\}. \tag{18}$$

We express now cot $h\rho$ in a series that is

$$\cot h\rho = \frac{1}{\rho} + \frac{\rho}{3} - \frac{\rho^3}{45} + \frac{2\rho^5}{945} \dots$$

and consequently we have in a similar manner as that for compression members,

$$\frac{1}{\rho \cot h\frac{\rho}{2} - 2} = \frac{6}{\rho^2} + \frac{1}{10} - \frac{\rho^3}{1400} \dots$$

$$m_t = \frac{4}{\rho} + \frac{2\rho}{15} - \frac{11\rho^3}{6300} \dots$$

$$n_t = \frac{2}{\rho} - \frac{\rho}{30} + \frac{13\rho^3}{12600} \dots$$

3. Determination of the Deflection Angles from the Conditions of Equilibrium.

The final step consists in writing down the equations from which the unknown deflection angles are to be computed, and in showing the manner in which these calculations are effected.

With the knowledge of the values of the deflection angles the problem is really solved, as the remainder of the computations

refers to simple operations with equations already known. With this end in view, we consider the equations:

$$\left.\begin{aligned} M_1 &= \frac{S}{T}\{m_c\tau_1 \pm n_c\tau_2\} \\ M_2 &= \frac{S}{T}\{\pm m_c\tau_2 + n_c\tau_1\} \end{aligned}\right\}\text{for compression members.}$$

$$\left.\begin{aligned} M_1 &= \frac{S}{T}\{m_t\tau_1 \pm n_t\tau_2\} \\ M_2 &= \frac{S}{T}\{\pm m_t\tau_2 + n_t\tau_1\} \end{aligned}\right\}\text{for tension members.}$$

As

$$\frac{S}{T} = \frac{S\sqrt{EI}}{\sqrt{S}} = \sqrt{EIS}, \text{ we write}$$

$$\left.\begin{aligned} m_c\frac{S}{T} &= K = \left\{\frac{4}{\rho} - \frac{2}{15}\rho - \frac{11}{6300}\rho^3 \dots\right\}\sqrt{EIS} \\ n_c\frac{S}{T} &= L = \left\{\frac{2}{\rho} + \frac{1}{30}\rho + \frac{13}{12600}\rho^3 \dots\right\}\sqrt{EIS} \end{aligned}\right\}\text{for compression members.}$$

$$\left.\begin{aligned} m_t\frac{S}{T} &= K = \left\{\frac{4}{\rho} + \frac{2}{15}\rho - \frac{11}{6300}\rho^3 \dots\right\}\sqrt{EIS} \\ n_t\frac{S}{T} &= L = \left\{\frac{2}{\rho} - \frac{1}{30}\rho + \frac{13}{12600}\rho^3 \dots\right\}\sqrt{EIS} \end{aligned}\right\}\text{for tension members.}$$

The general equation for the moment M_1, which is the moment at the end a of any of the bars in Figs. 14–19, is

$$M_1 = \frac{S}{T}\left\{m\tau_1 \pm n\tau_2\right\} = K\tau_1 \pm L\tau_2, \tag{19}$$

in which the letter c or t, pointing to compression or tension, has been omitted, in consequence of which m, n may refer to either a compression or tension member; it depends on what bar is under consideration, and with this understanding the equation will be used.

We now consider that in Fig. 21, the bar ends, formerly called a, and 4 in this particular case, all meet in the panelpoint o. We

give further to each moment the positive sign, when it turns in the same direction as that of a hand of a clock, as is shown by the arrows, which indicates that the angles of deflection are produced by clockwise revolution. If the result of the computation shows a

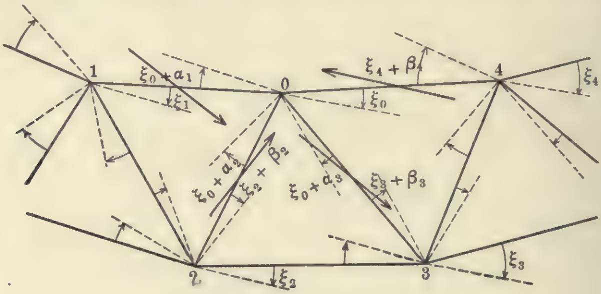


Fig. 21.

positive sign for a moment, then it turns as shown in the figure; and if a negative sign, it turns in the opposite direction.

The equilibrium of panelpoint o requires that

$$\sum_{1}^{4} M = 0. \tag{20}$$

If we substitute in equation (20) the 4 moments of the form of the equation (19) by using the notation as given in Fig. 21, where the angles of deflection τ appear either in the form

$$\xi + \alpha \text{ or } \xi + \beta,$$

then we have

$$\left.\begin{array}{l} K_{01}\{\xi_0 + \alpha_1\} + L_{01}\{\xi_1 + \beta_1\} \\ K_{02}\{\xi_0 + \alpha_2\} + L_{02}\{\xi_2 + \beta_2\} \\ K_{03}\{\xi_0 + \alpha_3\} + L_{03}\{\xi_3 + \beta_3\} \\ K_{04}\{\xi_0 + \alpha_4\} + L_{04}\{\xi_4 + \beta_4\} \end{array}\right\} = 0. \tag{21}$$

Calling

$$K_{01} + K_{02} + K_{03} + K_{04} = \Sigma K,$$
$$K_{01} \times \alpha_1 + K_{02} \times \alpha_2 + K_{03} \times \alpha_3 + K_{04} \times \alpha_4 = \Sigma K\alpha,$$
$$L_{01} \times \beta_1 + L_{02} \times \beta_2 + L_{03} \times \beta_3 + L_{04} \times \beta_4 = \Sigma L\beta,$$
$$L_{01} \times \xi_1 + L_{02} \times \xi_2 + L_{03} \times \xi_3 + L_{04} \times \xi_4 = \Sigma L\xi,$$

the four equations (21) are written,

$$\xi_0 \Sigma K + \Sigma K\alpha + \Sigma L\beta + \Sigma L\xi = 0,$$

or

$$\xi_0 = -\frac{\Sigma K\alpha + \Sigma L\beta + \Sigma L\xi}{\Sigma K}. \tag{22}$$

Each panelpoint gives an equation for a deflection angle ξ, and as we have just as many unknown ξ as panelpoints, the deflection angles ξ can be determined. But the computation of the angle ξ_0 from the equation (22) has a difficulty, which forbids its direct solution, and this difficulty consists in the fact that the third member of the numerator contains the four unknown quantities ξ_1, ξ_2, ξ_3, and ξ 4.

In order to remove the difficulty, we resort to trial computations, and put tentatively $\xi_1 = \xi_2 = \xi_3 = \xi_4 = 0$, and compute the angle ξ_0 under this supposition, which is in so far justified, as the $\Sigma L\xi$ is very small compared with the sum of the others.

In a similar way we proceed with the determination of the deflection angles ξ of the other panelpoints, and consequently also with that of ξ_1, ξ_2, ξ_3, and ξ_4, and substitute these latter values in equation (22), which now yields a more precise value of ξ_0, and in this way we continue the operations until satisfactory results are obtained. After these values for ξ are known they are substituted in equation (5), and then the deflection angles τ are computed, and finally the values of τ thus found are substituted in equation (19), whereupon the moments M are obtained. If d is the distance between the neutral axis and the extreme fiber, we find the secondary stress

$$\sigma = \frac{Md}{I}.$$

The position of the lines of direct stresses in the bars is an easy matter to determine after the values M are known. Considering that the angle ω, which is included between the resulting force P

and the chord of the deformed bar, is always very small, we commit no material error in putting $P = S$, so that we have,

$$f_1 = \frac{M_1}{S}, \quad \text{and } f_2 = \frac{M_2}{S}.$$

If we make ample provision against buckling in the design of the truss members, that is, if we design them with large moments of inertia, then $\rho = lT = l\sqrt{\frac{S}{EI}}$ becomes very small, and the equation (19), assuming a double curvature so that both deflection angles τ_1 and τ_2 are positive, is then written

$$M_1 = \left\{\frac{4}{\rho}\tau_1 + \frac{2}{\rho}\tau_2\right\}\sqrt{EIS},$$

and for M_2 we get

$$M_2 = \left\{\frac{4}{\rho}\tau_2 + \frac{2}{\rho}\tau_1\right\}\sqrt{EIS}.$$

After some simple transformations we find from these two equations the values of the deflection angles

$$\left.\begin{aligned} \tau_1 &= \frac{l\{2M_1 - M_2\}}{6EI}, \\ \tau_2 &= \frac{l\{2M_2 - M_1\}}{6EI}. \end{aligned}\right\} \tag{23}$$

These equations (23) are identical with the equations (2) in Chapter II, which latter we developed by applying the principle of the derivative of work.

The derivation of the equations (2) is based on the assumption that the deformation of a bar compared with its dimensions is very small and consequently a negligible quantity, but the foregoing analysis demonstrates that this assumption is only justified for large moments of inertia. Therefore, the equations (23) or (2) should not be employed for slim and flexible members.

CHAPTER IV.

MÜLLER-BRESLAU'S METHOD.

1. Derivation of the Fundamental Equations.

THIS method neglects the influence of the deformation of the bar on the secondary stresses, which is justified, as we have seen in Manderla's method, for sufficiently large moments of inertia.

The method proceeds from the assumption that the exterior loads, for which the secondary stresses are to be computed, are exclusively applied at the panelpoints and produce a deformation of the axis of each bar showing no point of inflection, as indicated in Fig. 22.

This assumed deformation, made for the sake of determining the character of the signs of the bending moments, does not exist

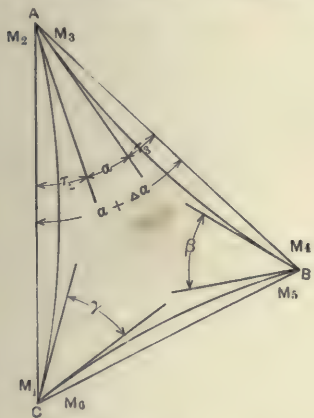


Fig. 22.

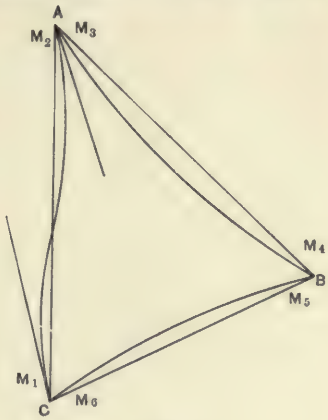


Fig. 23.

in reality, because the sum of the alterations of the angles α, β, and γ must vanish, that is to say, we must have

$$\Delta\alpha + \Delta\beta + \Delta\gamma = 0.$$

For instance, should the moment M_1 be found negative, then the deformation of the triangle is that of Fig. 23.

The alterations of the angles are to be calculated according to equations (1) in Chapter II, under the assumption of frictionless pins, but, as the bar ends are supposed to be rigidly fixed by riveting, the angle alterations are conceived as reduced by the bending moments to their original values α, β, and γ, as previously explained in Chapter II.

The equations (3) are those for the deflection angles of a bar with single curvature, and they are written with regard to the designations as given in Fig. 22,

$$\left.\begin{aligned} \tau_2 &= \frac{1}{6}\left\{M_1 + 2\,M_2\right\}\frac{l_1}{EI_1}, \\ \tau_3 &= \frac{1}{6}\left\{2\,M_3 + M_4\right\}\frac{l_2}{EI_2}, \end{aligned}\right\} \qquad (24)$$

in which l_1, l_2, I_1, I_2, are the lengths and moments of inertia of the bars AC and AB. M denotes a bending moment and E is the modulus of elasticity.

We have further,

$$\left.\begin{aligned} \tau_2 + \tau_3 &= \Delta\alpha, \\ \tau_4 + \tau_5 &= \Delta\beta, \\ \tau_1 + \tau_6 &= \Delta\gamma. \end{aligned}\right\} \qquad (25)$$

If we substitute the values τ_2, τ_3 . . . . from equations (24) in equations (25) we find

$$\left.\begin{aligned} \{M_1 + 2\,M_2\}\frac{l_1}{I_1} + \{2\,M_3 + M_4\}\frac{l_2}{I_2} &= 6\,E\Delta\alpha, \\ \{M_3 + 2\,M_4\}\frac{l_2}{I_2} + \{2\,M_5 + M_6\}\frac{l_3}{I_3} &= 6\,E\Delta\beta, \\ \{M_5 + 2\,M_6\}\frac{l_3}{I_3} + \{2\,M_1 + M_2\}\frac{l_1}{I_1} &= 6\,E\Delta\gamma. \end{aligned}\right\} \qquad (26)$$

If we assume for the present that three moments are known, we would then be in a position to calculate the other three bending moments from equations (26). Later illustrations will show the

manner in which the supposedly known moments are found. Now for the sake of convenience we do not introduce the unknown moments M in the computations, but the expressions

$$M\frac{l}{I} = U, \text{ and put}$$

for the bar $AC = l_1 : M_1\frac{l_1}{I_1} = U_1$ and $M_2\frac{l_1}{I_1} = U_2$.

" " " $AB = l_2 : M_3\frac{l_2}{I_2} = U_3$ and $M_4\frac{l_2}{I_2} = U_4$.

" " " $BC = l_3 : M_5\frac{l_3}{I_3} = U_5$ and $M_6\frac{l_3}{I_3} = U_6$.

With these expressions inserted in equations (26) we obtain

$$U_1 + 2\,\{U_2 + U_3\} + U_4 = 6\,E\Delta\alpha. \tag{27}$$
$$U_3 + 2\,\{U_4 + U_5\} + U_6 = 6\,E\Delta\beta. \tag{28}$$
$$U_5 + 2\,\{U_6 + U_1\} + U_2 = 6\,E\Delta\gamma. \tag{29}$$

We have further the relation

$$U_1 + U_2 + U_3 + U_4 + U_5 + U_6 = 0, \tag{30}$$

because the sum of the alterations of the angles α, β, and γ equals zero, that is, $\Delta\alpha + \Delta\beta + \Delta\gamma = 0$. We now express U_4, U_5 and U_6 as functions of the assumed known quantities U_1, U_2, and U_3, and obtain from equation (27),

$$U_4 = 6\,E\Delta\alpha - U_1 - 2\,\{U_2 + U_3\}, \tag{31}$$

and from equations (28) and (30),

$$U_5 = 6\,E\Delta\beta + U_1 + U_2 - U_4, \tag{32}$$

and from equations (29) and (30),

$$U_6 = 6\,E\Delta\gamma - U_1 + U_3 + U_4. \tag{33}$$

As soon as the quantities U are known, the bending moments M are also known, and with them, of course, the secondary stresses; but in order to find these stresses in a truss, it is necessary to apply the foregoing equations successively to the different triangles,

which compose the truss, whereby the triangles are supposed to be alternately deformed according to Fig. 22 or Fig. 24.

In order to calculate the alterations of the angles in those triangles deformed as in Fig. 24, we must bear in mind to reverse the signs of $\Delta\alpha$, $\Delta\beta$, and $\Delta\gamma$, and to write

$$\left.\begin{aligned} \{s_1 - s_3\} \cot \gamma + \{s_2 - s_3\} \cot \beta &= E\Delta\alpha, \\ \{s_2 - s_1\} \cot \alpha + \{s_3 - s_1\} \cot \gamma &= E\Delta\beta, \\ \Delta\alpha + \Delta\beta + \Delta\gamma &= 0. \end{aligned}\right\} \qquad (34)$$

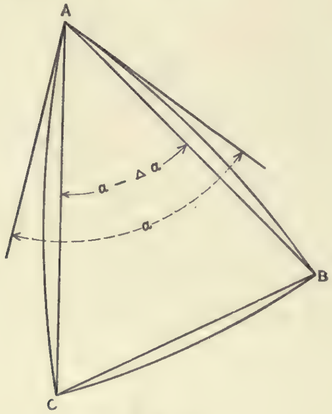


Fig. 24.

2. Determination of the Quantities U for a Truss.

In calculating the quantities U for a truss, Fig. 25, we assume for the present that the values U_1 and U_2 are known, and we then express every other U as a function of U_1 and U_2. The equilibrium requires that for every panelpoint $\Sigma M = 0$, consequently we have at panelpoint A:

$$M_2 - M_3 = 0, \text{ or } U_3 \frac{I_2}{l_2} = U_2 \frac{I_1}{l_1}.$$

After U_3 is known, its value is substituted in equation (31) and U_4 calculated, whereupon U_5 is found from equation (32) after the substitution of U_4, and in a similar way U_6 is found from equation (33).

At panelpoint B the value U_7 is determined from the condition that

$$M_6 - M_1 - M_7 = 0,$$

or

$$U_7 \frac{I_4}{l_4} = U_6 \frac{I_3}{l_3} - U_1 \frac{I_1}{l_1}.$$

In going over to the triangle called II in Fig. 25, we simply repeat the calculation made for triangle I, that is to say, we write down the equations for U_8, U_9, and U_{10} in conformity with equations (31), (32), and (33), care being taken that the alterations $\Delta\alpha$,

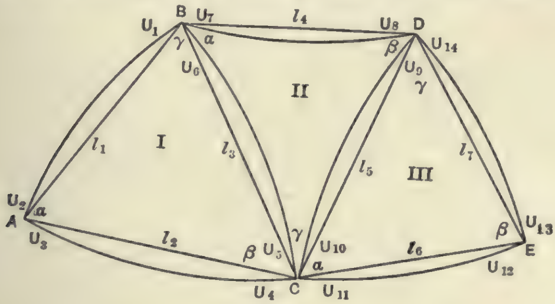


Fig. 25.

$\Delta\beta$ and $\Delta\gamma$ are calculated for triangle I after equations (34) and for triangle II after equations (1). Knowing U_4, U_5 and U_{10}, we find U_{11} at panelpoint C from

$$-M_4 - M_5 + M_{10} - M_{11} = 0,$$

and finally U_{12}, U_{13} and U_{14} are found from equations which again correspond to the fundamental equations (31), (32), and (33).

We have now expressed each U as a function of U_1 and U_2, which were supposed to be known, and the next step consists in determining U_1 and U_2. To this end we apply the condition $\Sigma M = 0$ to the panelpoints D and E, and write

$$\left.\begin{aligned} U_{14}\frac{I_7}{l_7} - U_9\frac{I_5}{l_5} + U_8\frac{I_4}{l_4} &= 0, \\ U_{12}\frac{I_6}{l_6} - U_{13}\frac{I_7}{l_7} &= 0. \end{aligned}\right\}$$

The solution of these equations gives U_1 and U_2, and herewith the problem is solved.

3. Method of Influence Lines.

The secondary stresses are usually computed for one single position of the loads. A more complete investigation would consist in the computation of these stresses under the assumption of full load and one or more partial loadings, but the most complete investigation applies the method of influence lines.

By Manderla's method we have seen that the values K and L are higher functions of the primary stresses S, and as the secondary stresses are dependent on K and L, they are also higher functions of S or of the exterior loads, in consequence of which the employment of the method of influence lines is not to be thought of.

It was also shown that if the influence of the deformations of the bars is left out of consideration, then the resulting equations are the more exact the larger the moments of inertia or the stiffer the truss members are, and in this case the secondary stresses are

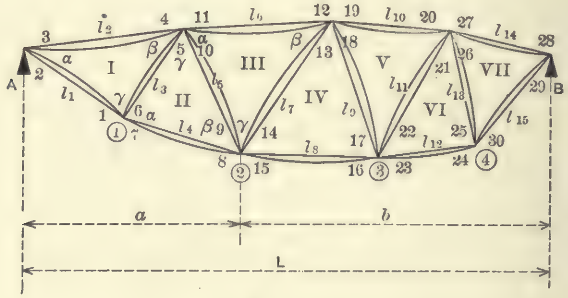


Fig. 26.

found to be linear functions of the exterior loads, and the method of influence lines is applicable.

In using this method, an influence line may be drawn for each bar end under the assumption of a traveling load equal to unity which is successively applied to the panelpoints of the truss.

Müller-Breslau's method of influence lines is in substance as follows:

Figure 26 represents a truss composed of the triangles I to VII, the sides of which show alternating deformations, and whose angle alterations must be calculated alternately in compliance with equations (1) and (34). The values U_1, U_2, U_3, etc., correspond to the figures 1, 2, 3, etc., written near the bar ends in Fig. 26.

The first step consists in the calculation of the angle alterations after a load is applied to the nearest panelpoint of the right-hand support — in our case panelpoint (4) — and of such magnitude that it produces at the left-hand support a reaction = 1. For this reaction = 1, each U value from U_3 to U_{27} inclusive is expressed as a function of U_1 and U_2 in a manner as has been previously explained. Thereupon the load is shifted from panelpoint (4) to panelpoint (1), and it is given such a magnitude that it produces at the right-hand support B a reaction = 1.

The calculations must now be repeated. First, all angle alterations are calculated for a reaction = 1 at B, and each U from U_{28} to U_4 is expressed as a function of U_{30} and U_{29}.

If the load is applied at (4) the general expression for U_m is

$$U_m = V_m + C_m U_1 + D_m U_2, \tag{35}$$

and if the load is applied at (1) we have

$$U_m = W_m + F_m U_{30} + G_m U_{29}. \tag{36}$$

V_m, W_m, C_m, D_m, F_m, and G_m are known quantities, and V_m and W_m alone are functions of the alterations of the angles which are dependent on the assumed exterior load.

If we now apply to panelpoint (2) a vertical load equal to unity, then this load produces a reaction at the left-hand support equal to $R_a = 1 \times \frac{b}{L}$, and the stresses due to this reaction in the sides of the triangles marked I and II are $\frac{b}{L}$ times greater than those pro-

duced by the reaction $R_a = 1$, in consequence of which the equation (35) is transformed into

$$U_m = \frac{b}{L} V_m + C_m U_1 + D_m U_2. \tag{37}$$

The value U_{11} is also computed after equation (37) for the simple reason that the condition $\Sigma M = 0$ must be fulfilled, that is to say, we must have

$$-M_{11} + M_{10} - M_5 + M_4 = 0,$$

or, by multiplying with -1 and writing $U\frac{I}{l}$ instead of M, we get

$$U_{11}\frac{I_6}{l_6} - U_{10}\frac{I_5}{l_5} + U_5\frac{I_3}{l_3} - U_4\frac{I_2}{l_2} = 0.$$

The same reasoning holds also true for the right-hand reaction R_b, the stresses and angle alterations in the triangles IV, V; VI, and VII, so that we are in a position to write down the following set of equations:

$$\left.\begin{aligned}
U_8 &= \frac{b}{L} V_8 + C_8 U_1 + D_8 U_2 \\
U_9 &= \frac{b}{L} V_9 + C_9 U_1 + D_9 U_2 \\
U_{10} &= \frac{b}{L} V_{10} + C_{10} U_1 + D_{10} U_2 \\
U_{11} &= \frac{b}{L} V_{11} + C_{11} U_1 + D_{11} U_2 \\
U_{12} &= \frac{a}{L} W_{12} + F_{12} U_{30} + G_{12} U_{29} \\
U_{13} &= \frac{a}{L} W_{13} + F_{13} U_{30} + G_{13} U_{29} \\
U_{14} &= \frac{a}{L} W_{14} + F_{14} U_{30} + G_{14} U_{29} \\
U_{15} &= \frac{a}{L} W_{15} + F_{15} U_{30} + G_{15} U_{29}
\end{aligned}\right\} \tag{38}$$

With respect to equations (32) and (30) and the condition $\Sigma M = 0$, applied to panelpoint (2), we now write down:

$$\left.\begin{aligned} &U_{10} + U_{11} = 6\,E\Delta\alpha_{\mathrm{III}} + U_{13} + U_{14}, \\ &U_{12} + U_{13} = 6\,E\Delta\beta_{\mathrm{III}} + U_9 + U_{10}, \\ &U_9 + U_{10} + U_{11} + U_{12} + U_{13} + U_{14} = 0 \\ &-U_{15}\frac{I_8}{l_8} + U_{14}\frac{I_7}{l_7} - U_9\frac{I_5}{l_5} + U_8\frac{I_4}{l_4} = 0. \end{aligned}\right\} \qquad (39)$$

After the values U_8 to U_{15}, as expressed in equations (38), are inserted in equations (39), they can be solved for U_1, U_2, U_{30} and U_{29}. In applying the load = 1 in succession to the different panelpoints of the truss, we obtain by the same course of treatment the influence lines for U_1, U_2, U_{30} and U_{29}, and with these latter values the influence lines for any U.

We caution the reader to observe the fact that $\Delta\alpha_{\mathrm{III}}$ and $\Delta\beta_{\mathrm{III}}$ in equations (39) require a separate calculation for the reason that, if the load = 1 is supposed at (2), then the reaction $R_a = 1 \times \frac{b}{L}$ produces $\frac{b}{L}$ times greater stresses and angle alterations than $R_a = 1$ in the triangles I and II, but not in triangle III.

For each U we have two equations, the use of which depends on the position of the load. If, for instance, the load is applied either at panelpoint (3) or (4), then

$$U_{13} = \frac{b}{L}V_{13} + C_{13}U_1 + D_{13}U_2;$$

but if the load is at (1) or (2), the equation

$$U_{13} = \frac{a}{L}W_{13} + F_{13}U_{30} + G_{13}U_{29}$$

must be used.

CHAPTER V.

RITTER'S METHOD.

OUR first step will consist in assigning the proper notation for the different quantities with which we have to deal.

In Fig. 27, representing a part of a truss, we will for the present exclusively consider panelpoint (5) where four bars intersect, forming three angles. Each of these angles included between any

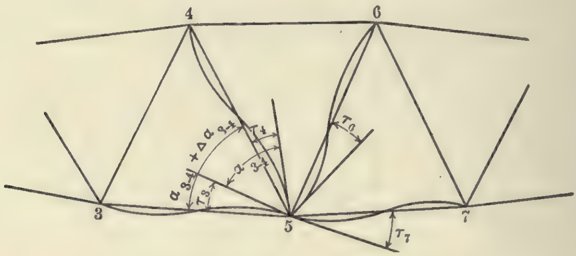


Fig. 27.

two adjacent straight bars will be denoted by an index corresponding to the two figures of the opposite bar. So, for instance, the $\sphericalangle$ 3–5–4 shall have the index 3–4, the $\sphericalangle$ 4–5–6 the index 4–6, etc.

The two bending moments of the bar 3–5 will be designated M_3 at panelpoint (5) and M_3' at panelpoint (3). For the bar 4–5 the moments shall be M_4 at panelpoint (5) and M_4' at panelpoint (4), etc. But in case we consider panelpoint (4) the designations of the two moments of the bar 4–5 are M_5 at panelpoint (4) and M_5' at panelpoint (5). Consequently we can write:

$$M_4 \text{ for panelpoint } (5) = M_5' \text{ for panelpoint } (4),$$
$$M_4' \text{ for panelpoint } (5) = M_5 \text{ for panelpoint } (4),$$

which means that each moment in a truss is characterized in two different ways. A moment will be taken as positive when it deflects a bar in the sense of the hand of a clock.

If each bar in our truss could turn around a frictionless pin, then each angle α included between any two adjacent bars would be changed under the influence of the actual loading of the truss; these angles would be either increased or decreased an amount $\Delta\alpha$ corresponding to the alterations in the lengths of the bars. But the bar ends of our truss are riveted, consequently the angle included between any two adjacent end tangents drawn to the curves of the deformed bars, which we assume to be S-shaped, must remain unchanged during deformation, and this unchangeable angle is α. We refer the reader here to Fig. 1 in Chapter II, and to what has been further said on this subject.

Figure 27 shows that

$$\alpha_{3-4} + \Delta\alpha_{3-4} + \tau_4 = \alpha_{3-4} + \tau_3,$$

or

$$\Delta\alpha_{3-4} = \tau_3 - \tau_4.$$

Substituting the values of τ_3 and τ_4 as given in equations (2) of Chapter II, we have:

$$E\Delta\alpha_{3-4} = \frac{l_3\{2\,M_3 - M_3'\}}{6\,I_3} - \frac{l_4\{2\,M_4 - M_4'\}}{6\,I_4}\,.$$

Similar equations can be written for $E\Delta\alpha_{4-6}$ and $E\Delta\alpha_{6-7}$.

We put now for the sake of convenience $\dfrac{lM}{6\,I} = U$, U representing a force per unit of area and measured with the same unit as s and $E\Delta\alpha$. The equilibrium at a panelpoint requires that the algebraic sum of the moments vanishes, or that

$$M_3 + M_4 + M_6 + M_7 = 0,$$

and the four equations in regard to panelpoint (5) are now as follows:

$$\left.\begin{aligned}
E\Delta\alpha_{3-4} &= \{2\,U_3 - U_3'\} - \{2\,U_4 - U_4'\},\\
E\Delta\alpha_{4-6} &= \{2\,U_4 - U_4'\} - \{2\,U_6 - U_6'\},\\
E\Delta\alpha_{6-7} &= \{2\,U_6 - U_6'\} - \{2\,U_7 - U_7'\}.\\
\frac{I_3}{l_3}\,U_3 + \frac{I_4}{l_4}\,U_4 &+ \frac{I_6}{l_6}\,U_6 + \frac{I_7}{l_7}\,U_7 = 0.
\end{aligned}\right\} \qquad (40)$$

For every panelpoint there are as many equations as bars, which intersect at the panelpoint, or two equations for each bar, so that the total number of equations equals the number of unknown moments.

The work of solving a larger set of equations algebraically taxes the patience of a quick and sure computer, experienced in all the short cuts that can be advantageously used, and any means which are calculated to save time are very welcome indeed.

For this reason we will now consider a graphical solution of the problem, the object being to find the values U.

The $E\Delta\alpha$ are computed by means of equations (1) in Chapter II, and are due to the actual loading of the truss, for which loading we intend to determine the secondary stresses. If we assume for the sake of an illustration that besides the $E\Delta\alpha$ also the values of U' were known, we could then easily and quickly find the values U by the simple means of a force and string polygon in the following manner:

We consider the values $\frac{I}{l}$ as forces, lay out a vertical load line, select an arbitrary pole O_5 with the pole distance H_5, and draw the rays as in Fig. 28. Hereupon the distances between the forces, which are the known $E\Delta\alpha$, are laid out horizontally as in Fig. 29, and each of the forces displaced to the left an amount U' and the equilibrium polygon constructed for the forces thus displaced.

The double values of U are now equal to the distances of the displaced forces from their resultant, a statement which follows from the equations (40).

The distance of a displaced force from the resultant is $2\,U$ for a displacement U', consequently the distance of an undisplaced

force from the resultant equals 2 $U-U'$, and the difference between the distances of two adjacent undisplaced forces must equal $E\Delta\alpha$.

The meaning of this is that the first three of the equations (40)

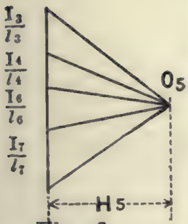


Fig. 28.

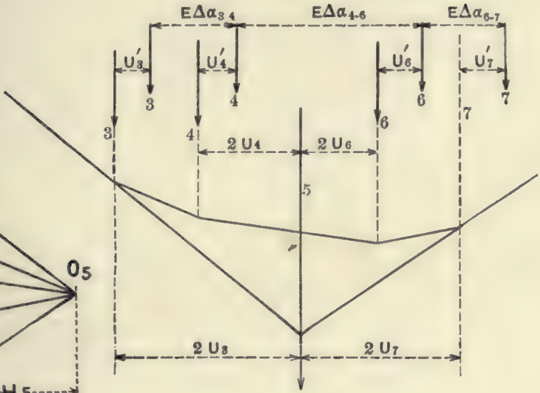


Fig. 29.

have been satisfied, and since the string polygon shows that the algebraic sum of the component moments in reference to any point in the direction of the resultant vanishes, the last equation is also satisfied.

As a matter of fact, the values of U' are not known at all, and herein of course lies an obstacle to our solution, but this can be removed by trials.

Considering the fact that any change in the values of U' has only half the effect on the values of U, we will for the first trial assume that the quantities U' are non-existent, in which case we obtain roughly approximate values U; or, in other words, for the first trial we will assume that the forces are not at all displaced and with this understanding we draw a force and equilibrium polygon for every panelpoint.

Figures 30–31 show the positions of the resultants of the undisplaced forces for the panelpoints (4) and (5), the equilibrium polygons being omitted.

If now the distance between an undisplaced force and the resultant is designated by V, we have, with reference to Figs. 30–31,

$$2\,U_5 = U_5' + V_5,$$
$$2\,U_4 = U_4' + V_4.$$

In compliance with the designations as adopted at the beginning, we have also

$$U_5' = U_4,$$
$$U_4' = U_5.$$

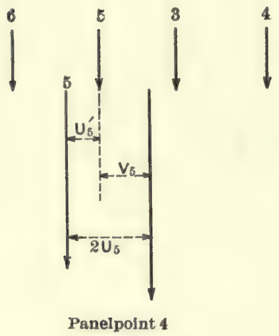


Fig. 30.

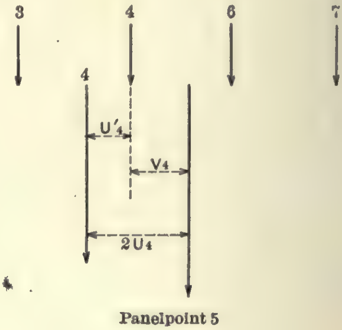


Fig. 31.

Further,

$$U_5' = \frac{U_4' + V_4}{2} = \frac{U_5 + V_4}{2} = \frac{U_5' + V_5}{4} + \frac{V_4}{2},$$

or

$$U_5' = \frac{V_5 + 2\,V_4}{3},$$

and in a similar way we find

$$U_4' = \frac{V_4 + 2\,V_5}{3}.$$

The determination of the quantities U would be an easy matter if the positions of the resultant forces were known, in which case they could be found at once; but since these positions are not known and must be first found, we take for instance U_5, which equals

$$U_5 = U_4' = \frac{V_4 + 2\,V_5}{3},$$

and transfer it as U_4' in the equilibrium polygon for panelpoint (5); and in a similar way we transfer any other U as U', which has been obtained by a first trial, into some other and corresponding equilibrium polygon, and continue these correcting operations until the changes in the values are so small that they can be neglected, but we must bear in mind that each equilibrium polygon has to be repeatedly drawn.

After the quantities U are known, we obtain the bending moments from the equations:

$$M = \frac{6\,I}{l}\,U,$$

$$M' = \frac{6\,I}{l}\,U';$$

and if d is the distance from the center line of gravity to the extreme fiber, and σ the secondary stress per square unit, then

$$\sigma = \frac{Md}{I} = \frac{6\,d}{l}\,U.$$

Strict attention must be paid to the character of the signs in order to avoid mistakes. The succession of the bars around a panelpoint should be taken in the sense of motion of the hand of a clock, and a $\Delta\alpha$, which has a positive sign, should be laid out on the right hand, and a negative $\Delta\alpha$ on the left hand. Any quantity U to be transferred as U' into a corresponding equilibrium polygon should be laid out to the left, if it is situated to the left of the resulting force of the $\frac{I}{l}$, and it should be laid out to the right, if on the right side of the resultant.

The method we have explained is in so far approximate as the influence of the deformation on the secondary stresses, due to the action of a force along the chord of the deformed bar, has not been taken into account; and if this longitudinal force is also to be considered, it is necessary to develop an expression for the deflection angle τ differing from that given in equations (2) in Chapter II.

The equation of the elastic line with the designations as given in Fig. 32 is written

$$\frac{d^2y}{dx^2} = - \frac{M_x}{EI},$$

M_x being the moment at any arbitrary point of the axis of the bar with the coördinates x, y.

If it is now supposed that the bar is acted upon not only by the

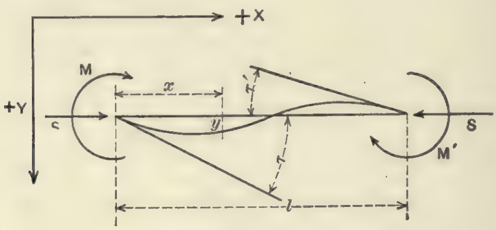


Fig. 32.

two end bending moments M and M', but also by a compressive force of the magnitude S, then

$$M_x = Sy + \frac{M\{l - x\}}{l} - \frac{M'x}{l}.$$

Substituting this value M_x in the differential equation, we get

$$\frac{d^2y}{dx^2} = - \frac{Sy}{EI} + \frac{\{M + M'\}\, x - Ml}{EIL}.$$

The integration gives

$$y = \frac{M}{S} \cos \frac{Kx}{l} - \frac{M \cos K + M'}{S \sin K} \sin \frac{Kx}{l} + \frac{\{M + M'\} x - Ml}{Sl}.$$

In this equation the constants have been determined from the condition that y vanishes when $x = 0$ and when $x = l$, and K denotes the expression $\sqrt{\frac{Sl^2}{EI}}$ for brevity's sake.

The first differential coefficient gives the angle of deflection τ for $x = 0$, that is,

$$\frac{dy}{dx} = \tau = -\frac{K\{M \cos K + M'\}}{Sl \sin K} + \frac{M + M'}{Sl}.$$

Developing $\cos K$ and $\sin K$ in series gives

$$\tau = \frac{l\{2\,RM - R'M'\}}{6\,EI}, \tag{41}$$

and in this last equation

$$R = 1 + \frac{K^2}{15} + \frac{2\,K^4}{315} + \frac{K^6}{1575} + \ldots .$$

$$R' = 1 + \frac{7\,K^2}{60} + \frac{31\,K^4}{2520} + \frac{127\,K^6}{100800} + \ldots .$$

If M and M' are exchanged τ' will be found. Should the bar under consideration be a tension member, then the sign of K^2 in the equation $K^2 = \frac{Sl^2}{EI}$ must be reversed and R and R' computed accordingly, that is to say, every second member is negative.

For an infinitely great moment of inertia K^2 is reduced to zero and $R = R' = 1$, which leads us back to the equations (2) in Chapter II.

From this consideration we conclude that the equations (2) are the more exact the larger the moments of inertia are, or, in other words, they should only be used for stiff truss members which have ample provision against buckling, an assertion previously made and which has now been demonstrated to be true.

Should the more exact method be used in connection with graphic statics, it is necessary in constructing the force polygon to lay off the forces $\frac{I}{lR}$ and displace them in the equilibrium polygon an amount $R'U'$, whereupon the values $2\,RU$ are found instead of $2\,U$.

CHAPTER VI.

MOHR'S METHOD.

1. Determination of the Unknown Quantities.

In this method of calculation the effect of deformation on the leverarms y, as also that of the transverse forces Q, are not considered (see Fig. 2 in Chapter II), for reasons previously given, and only the bending moment at each end of the bar is determined.

If for a truss, which is composed of triangles, p denotes the number of panelpoints and n the number of bars, then

$$n = 2p - 3;$$

and as 2 unknown moments correspond to each bar, we have as the total number of unknown moments,

$$2n = 4p - 6.$$

But instead of introducing the unknown bending moments in the calculation, Mohr introduces two sets of angles on which the bending moments are dependent, and in so doing he reduces the total number of unknowns to $3p - 3$.

Figure 33 shows the unknown angles ϕ_0, ϕ_1, and ψ_{01} for the bar 01. The lines $0a$ and $1b$ are parallel to each other, and indicate the original direction of the bar 01, which is that before the exterior forces began to act. $0T_0$ and $1T_1$ indicate the end tangents drawn to the elastic line of the curved bar after deformation has set in. The angle ϕ_0 included between the original direction $0a$ of the bar and its end tangent $0T_0$ is constant for each bar end around the panelpoint 0 during deformation, or, in other words, ϕ_0 is the angle around which the end of each bar revolves during deformation. This fact follows from our assumption that all bar ends are rigidly riveted so that an angle between any two adjacent

bar ends must remain constant while a bar becomes deformed. The angle ψ_{01} included between the original direction o a or 1 b of the bar and the chord of the curved bar after deformation is the angle around which a bar revolves during deformation under the assumption that the truss is provided with frictionless pins.

To each panelpoint of the truss corresponds an angle ϕ, and as the number of the panelpoints is p, we have p as the total number

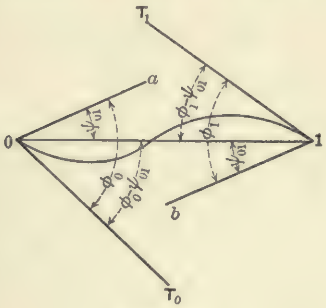


Fig. 33.

of the angles ϕ; to each bar corresponds an angle ψ, and as the number of the bars is $2p - 3$, we have $2p - 3$ as the total number of the angles ψ, consequently the number of angles ϕ and ψ taken together is equal to $p + 2p - 3 = 3p - 3$, which are now the number of unknowns of our problem.

2. Determination of the Angles ψ.

These angles ψ are calculated by the equation in Chapter II, $M\psi = \Sigma s\Delta l$, where $M = 1$, so that

$$1 \times \psi = \Sigma s \Delta l$$

for the assumption that the points of supports are fixed in the direction of the reactions. This is the case with respect to Fig. 34, which shows a Pratt truss with one fixed end and one roller end. If we wish, for instance, to determine the angle of revolution ψ for the diagonal in the third panel from the left end, a force $P = \frac{1}{l}$ must

be applied at each end of the diagonal and at right angles to its axis, which gives a moment $= + 1$. This moment produces a downward reaction $= \frac{1}{L}$ at the left end of the truss, and an upward reaction of the same magnitude at the right end. Hereupon the

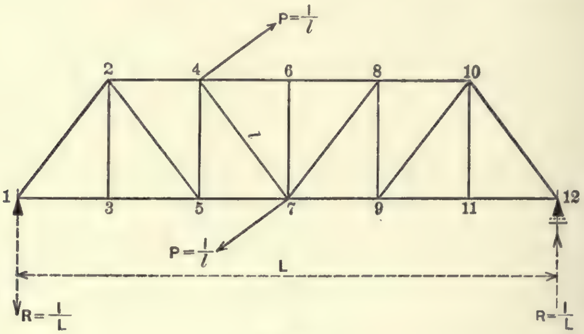


Fig. 34.

stresses s for each bar produced by the moment $= + 1$ and the reactions $\frac{1}{L}$ are calculated and the sum of the products $s\Delta l$ is formed, where $\Delta l = \frac{Sl}{EA}$, S being the stress produced by the actual loading (for example, by a railroad train), E being the modulus of elasticity and A the gross area of a bar.

3. The Bending Moments Expressed as Functions of the Angles ϕ and ψ.

In Chapter II, it was shown that the elastic line, whose equation is $\frac{d^2y}{dx^2} = \frac{M}{EI}$, can be conceived as an equilibrium curve, if we put, for instance, $EI = H$, and $M = p$; that is to say, by considering first the term EI as a force acting parallel along the chord of the deformed bar, and considering secondly the bending moment M as a force per unit of length of the equilibrium curve and at right angles to the chord of the deformed bar.

In order to obtain at once the proper signs for the moments M and the angles $\phi - \psi$, it is suitable for the purpose in view to give these moments and angles the positive signs, letting the moments turn in the sense like the hand of a clock, and producing the angles by clockwise revolution of a right line, as is indicated in Fig. 35.

The meaning of the angles $\phi - \psi$ is the same as previously explained with reference to Fig. 33. These angles are, of course,

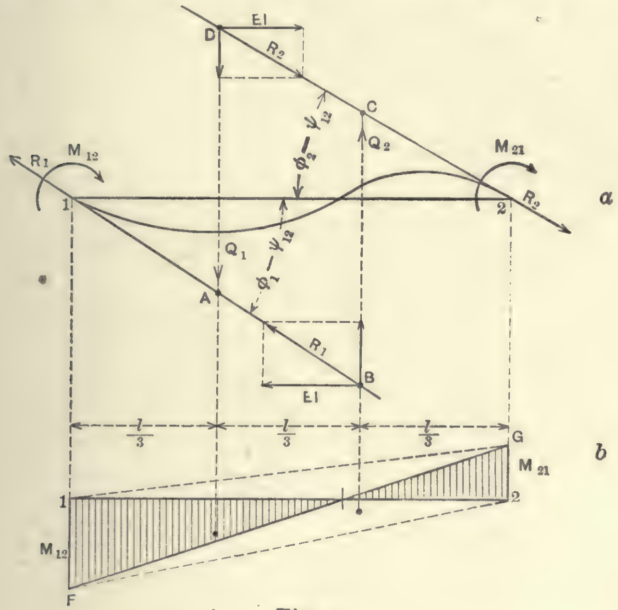


Fig. 35.

very small, but they are shown in the sketch largely exaggerated for the sake of a better illustration. The variable moments, considered as vertical forces to whose action the bar is subjected, are represented as the difference between the two triangles $1 - 2 - F$, and $F - G - 2$. Their resultant forces Q_1 and Q_2 pass through the centers of gravity of the triangles, distant one third of the length of the bar from each end. Besides the vertical forces Q_1 and Q_2 we have the longitudinal force EI acting upon the bar.

The four forces R_1, R_2, Q_1 and Q_2 are in equilibrium and form

the trapezoid $ABCD$. The equilibrium requires that the algebraic sum of the moments with respect to any point must be equal to zero. In taking the moments around the two points A and C, we have then the two equations,

$$\left.\begin{aligned} EI \times AD - Q_2\frac{l}{3} &= 0, \\ EI \times BC - Q_1\frac{l}{3} &= 0. \end{aligned}\right\} \qquad (42)$$

The lengths of the lines AD and BC, bearing in mind that the angles ϕ and ψ are very small, we find,

$$AD = \frac{l}{3}\{\phi_1 - \psi_{12}\} + \frac{2\,l}{3}\{\phi_2 - \psi_{12}\} = \frac{l}{3}\{\phi_1 + 2\,\phi_2 - 3\,\psi_{12}\},$$

$$BC = \frac{l}{3}\{\phi_2 - \psi_{12}\} + \frac{2\,l}{3}\{\phi_1 - \psi_{12}\} = \frac{l}{3}\{2\,\phi_1 + \phi_2 - 3\,\psi_{12}\}.$$

The forces Q_1 and Q_2 are

$$Q_1 = M_{12}\frac{l}{2} \text{ and } Q_2 = M_{21}\frac{l}{2}\cdot$$

Substituting the values of AD, BC, Q_1 and Q_2 in the equations (42), we have

$$\left.\begin{aligned} EI\frac{l}{3}\{\phi_1 + 2\,\phi_2 - 3\,\psi_{12}\} - M_{21}\frac{l^2}{6} &= 0, \\ EI\frac{l}{3}\{2\,\phi_1 + \phi_2 - 3\,\psi_{12}\} - M_{12}\frac{l^2}{6} &= 0. \end{aligned}\right\} \qquad (43)$$

Dividing each equation by $\frac{l^2}{6}$ and putting $\frac{2\,EI}{l} = N_{12}$, the equations (43) are then written,

$$\left.\begin{aligned} M_{12} &= N_{12}\{2\,\phi_1 + \phi_2 - 3\,\psi_{12}\}, \\ M_{21} &= N_{12}\{\phi_1 + 2\,\phi_2 - 3\,\psi_{12}\}. \end{aligned}\right\} \qquad (44)$$

In order to find the moments M it is necessary to determine next the unknown angles ϕ in equations (44); the values N are known, and the angles ψ have previously been calculated. As the algebraic sum of the moments with respect to any point must be equal

to zero, we take in succession the panelpoints of a truss as the centers of moments, and write down as many equations as there are panelpoints in the truss, which number equals the number of unknown angles ϕ.

With reference to Fig. 34, these equations are as follows:

$$\left.\begin{array}{lll}
\text{Panelpoint} & 1: & M_{1-2} + M_{1-3} = 0. \\
\text{``} & 2: & M_{2-1} + M_{2-3} + M_{2-5} + M_{2-4} = 0. \\
\text{``} & 3: & M_{3-1} + M_{3-2} + M_{3-5} = 0. \\
\text{``} & 4: & M_{4-2} + M_{4-5} + M_{4-7} + M_{4-6} = 0. \\
\text{``} & 5: & M_{5-3} + M_{5-2} + M_{5-4} + M_{5-7} = 0. \\
\text{``} & 6: & M_{6-4} + M_{6-7} + M_{6-8} = 0. \\
\text{``} & 7: & M_{7-5} + M_{7-4} + M_{7-6} + M_{7-8} + M_{7-9} = 0. \\
\text{``} & 8: & M_{8-6} + M_{8-7} + M_{8-9} + M_{8-10} = 0. \\
\text{``} & 9: & M_{9-7} + M_{9-8} + M_{9-10} + M_{9-11} = 0. \\
\text{``} & 10: & M_{10-8} + M_{10-9} + M_{10-11} + M_{10-12} = 0. \\
\text{``} & 11: & M_{11-9} + M_{11-10} + M_{11-12} = 0. \\
\text{``} & 12: & M_{12-10} + M_{12-11} = 0.
\end{array}\right\} \quad (45)$$

The values of the bending moments M as expressed in equations (44) are now substituted in equations (45), and the values of the angles ϕ are ascertained by solving the latter equations.

The last step consists in substituting the values ϕ in the equations (44), from which now the moments M can be calculated, and herewith our problem is solved.

Mohr suggests a short cut in regard to the solution of equations (45), provided the truss to be examined is symmetrical, in which case the conditions of equilibrium appear in symmetrical form. For a symmetrical truss the work of solving the equations is now greatly facilitated by determining first the sums and the differences of the angles ϕ, whose positions are symmetrical in respect to the center line of the truss. In our particular case we would determine first

$$\{\phi_1 + \phi_{12}\} \text{ and } \{\phi_1 - \phi_{12}\},$$
$$\{\phi_2 + \phi_{10}\} \text{ and } \{\phi_2 - \phi_{10}\},$$
$$\{\phi_3 + \phi_{11}\} \text{ and } \{\phi_3 - \phi_{11}\}, \text{ etc.}$$

CHAPTER VII.

METHOD OF LEAST WORK.

THERE is no doubt that any subject gains in clearness if looked at from different points of view, and for this reason we will make use of the principle of least work with which the reader is supposed to be familiar and apply it to the simplest truss, composed of three bars and shown in Fig. 36.

TABLE I.

Bar.	Length in Inches.	Stresses in Tons, if Pin Connected.		Area in Square Inches.	Gross Moment of Inertia, I, Reduced to Inches.	$\frac{I}{\text{length}}$.
		Total.	Per Square Inch.			
AC	400	-176.75	-6	29.46	800	2
AB	565.68	$+125.00$	$+7$	17.86	600	1.06
BC	400	-176.75	-6	29.46	800	2

The riveted truss is symmetrical in its form, loaded with a single concentrated load of 250 tons at its apex, and assumed to have one fixed and one movable end. The necessary data for the calculations are given in Table I, which are self-explanatory. Each bar is box-shaped, consisting of two 15-inch webs and 4 angles. Before analyzing the stresses, it is necessary to consider the nature of the truss under consideration.

We have seen that the lines of stresses in each bar of a riveted truss are displaced under the action of the exterior forces. Of these stresses we know neither their magnitude nor their position and direction; that is to say, each bar represents three unknown

quantities. If p designates the number of panelpoints and n the number of bars in a truss, then $n = 2p - 3$, and consequently the number of unknowns is equal to $3n = 6p - 9$. The statics of rigid bodies gives us three equations for each panelpoint, so that we have a total of $3p$ equations; but as this number includes three equations which refer to the equilibrium of the outer forces, and are

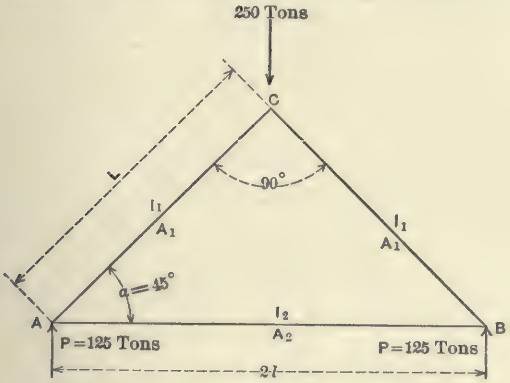


Fig. 36.

consequently of no use for our purpose to determine the inner forces, there are only $3p - 3$ equations available for the determination of the $6p - 9$ unknowns. The remainder of the equations, that is, $\{6p - 9\} - \{3p - 3\} = 3p - 6$, must therefore be obtained from some other source than statics. From the foregoing remarks we see that a triangular riveted truss is threefold statically indeterminate, a fact which can also be arrived at in a way different from the above.

The influence of the sectional areas on the stresses, which always exists in a statically indeterminate structure, can easily be detected in our truss by going to extremes. Let us suppose that the truss is loaded at its apex with a finite load, but that the moments of inertia of the two compression members are infinite, then any deformation of these members is excluded, and as the angle at the apex — the truss being riveted — remains unchanged, it follows that the length of the horizontal bar is not affected at all by the

exterior load. This is merely another way of stating that the horizontal bar is under no stress. If we now suppose each moment of inertia of the two compression members to be equal to zero, in which case each of these two members is represented by a very thin bar, hinged at its ends, then the horizontal bar receives a pull of 125 tons, but no bending moment. As neither of these conditions can be fulfilled, that is, as the moments of inertia of the members must be between 0 and ∞, we draw the conclusion that the pull in the horizontal bar is less than 125 tons.

The circumstances that the truss is symmetrical in form and also symmetrically loaded reduces the three unknowns to two unknowns. Generally speaking, we are free to select the unknowns of the problem, and in our case we take the pull of the horizontal bar and the bending moment at its middle as the two unknown quantities to be determined. The only stresses we need to consider in our case are direct and bending stresses; the shearing stresses, which may be taken into account, are in so far of no consequence, as their influence on the final result is small enough to be neglected, and the effect of the moments Sy, owing to the small deformation of the bars, we will also leave out of consideration.

The principle of least work requires the work of deformation of the truss to be a minimum, which means that the partial differential coefficients with respect to the unknowns must be placed each equal to zero. Therefore, we write the equations of condition for our particular case,

$$\int \frac{M}{EI} \frac{\partial M}{\partial U} dx + \int \frac{N}{EA} \frac{\partial N}{\partial U} dx = 0.$$

In this equation

M = moment with respect to any point of the axis of any bar,
N = direct stress in any bar,
I = moment of inertia of any bar,
A = sectional area of any bar,
E = constant modulus of elasticity,
U = any unknown.

As the truss is symmetrical in form and symmetrically loaded, it suffices to extend the work of deformation over one half of the truss, instead of over the entire truss.

If we now pass a cut through its middle, Fig. 37, apply the inner

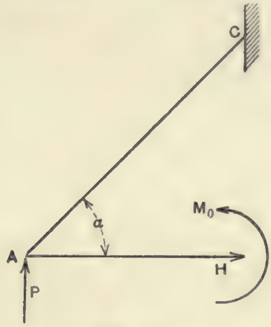


Fig. 37.

forces as outer forces in order to establish the equilibrium, take A as the origin of the abscissæ x, coincident with the axis AC and of the abscissæ v, coincident with the axis of the horizontal bar, resolve P and H in components parallel and at right angles to AC, we can then write, H and M_0 being the unknowns,

$$\left.\begin{aligned} M &= P\cos\alpha \times x - H\sin\alpha \times x - M_0, \\ \frac{\partial M}{\partial H} &= -\sin\alpha \times x;\ \frac{\partial M}{\partial M_0} = -1, \\ \text{and} \qquad & \\ N &= P\sin\alpha + H\cos\alpha, \\ \frac{\partial N}{\partial H} &= \cos\alpha;\ \frac{\partial N}{\partial M_0} = 0, \end{aligned}\right\} \text{for bar } AC.$$

$$\left.\begin{aligned} M &= -M_0, \\ \frac{\partial M}{\partial H} &= 0;\ \frac{\partial M}{\partial M_0} = -1, \\ \text{and} \qquad & \\ N &= H, \\ \frac{\partial N}{\partial H} &= 1;\ \frac{\partial N}{\partial M_0} = 0, \end{aligned}\right\} \text{for bar } AB.$$

By substituting these values in the equation of condition, we obtain, with respect to Fig. 36 and 37 two equations from which the two unknowns can be found.

These equations are,

$$\left.\begin{array}{l} -\int_0^L \frac{P\cos\alpha\sin\alpha^2}{I_1}dx+\int_0^L\frac{H\sin\alpha^2\times x^2}{I_1}dx+\int_0^L\frac{M_0\sin\alpha\times x}{I_1}dx \\ +\int_0^L\frac{P\cos\alpha\sin\alpha}{A_1}dx+\int_0^L\frac{H\cos\alpha^2}{A_1}dx+\int_0^l\frac{H}{A_2}dv=0 \\ -\int_0^L\frac{P\cos\alpha\times x}{I_1}dx+\int_0^L\frac{H\sin\alpha\times x}{I_1}dx+\int_0^L\frac{M_0}{I_1}dx+\int_0^l\frac{M_0}{A_2}dv=0. \end{array}\right\}$$

Computing these integrals and solving for H and M_0, we get

$$H=P\times\frac{\{a-b-c\}}{\{a-b+c+\varepsilon\}}$$

and

$$M_0=P\times\frac{\left\{\frac{a-c}{a+c+\varepsilon}-1\right\}}{\frac{d}{a+c+\varepsilon}-\frac{\frac{1}{I_1}+\varepsilon}{d}}$$

where $a=\frac{L^2}{6\,I_1}$, $b=\frac{L^3I_2}{8\,\{LI_1I_2+lI_1^2\}}$,

$c=\frac{1}{2\,A_1}$, $d=\frac{\cos\alpha L}{2\,I_1}$, and $\varepsilon=\frac{l}{LA_2}$. As $\alpha=45°$, each $\sin\alpha$ has been replaced by $\cos\alpha$.

If we assume $I_1=\infty$, we obtain

$$H=0,\text{ and }M_0=0;$$

and if we assume $I_1=0$, we get

$$H=P,\text{ and }M_0=0,$$

which are the same results as stated before. But if we substitute the values as given in Table I, we find the horizontal pull:

$$H = 124.55 \text{ tons},$$

and the moment

$$M_0 = M_3 = -32.8 \text{ inch-tons} = -65{,}600 \text{ inch-pounds},$$
$$M_3 = M_2 = M_4 = M_5. \quad \text{See Fig. 39.}$$

M_3 has the same direction of revolution as indicated in Fig. 37, and M_2 has the opposite direction of M_3.

The moment at the apex is

$$M_1 = Pl - H \sin \alpha \times L - 32.8,$$

or

$$M_1 = +94.47 \text{ inch-tons} = 188{,}940 \text{ inch-pounds.}$$

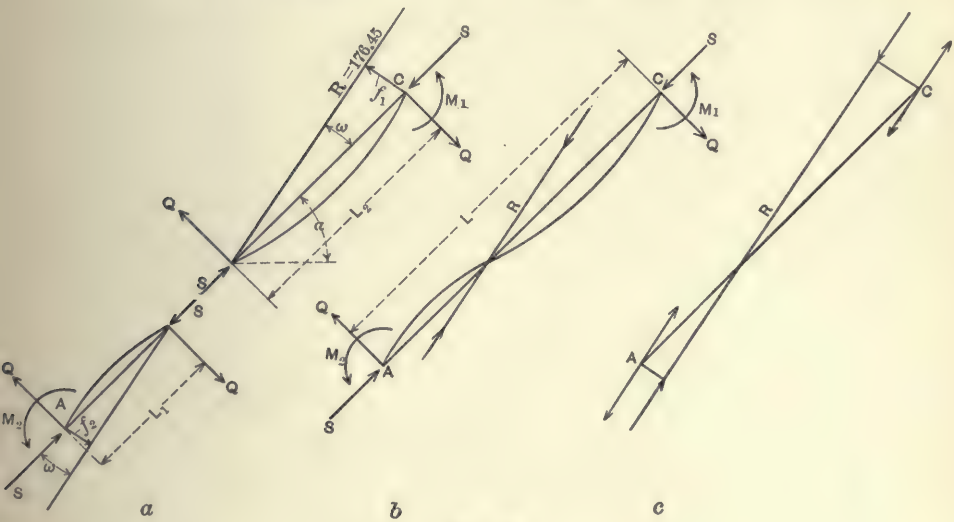


Fig. 38.

This moment corresponds to a stress of 1020 pounds per square inch in the outer fiber, and is not more than 8.5 per cent of the primary stress.

The stress in each compression member is $R = \sqrt{124.55^2 + 125^2} = 176.45$ tons.

This stress must pass through the point of inflection, a point where no moment exists. The fact that the horizontal pull is less than 125 tons points to the displacements of the directions of the compressive stresses as indicated in Fig. 38.

Passing a section through each end point of the bar AC and one through the point of inflection, and applying the stresses R as exterior forces, we can then replace R by $S = R \cos \omega$ and $Q = R \sin \omega$, a moment M_1 at point C and a moment M_2 at A. The equilibrium requires the identity in magnitude of the stresses S, acting along the chord of the deformed bar and the transverse forces Q at right angles to this chord; further, we must have

$$QL_2 = M_1 \text{ and } QL_1 = M_2, \text{ or } QL = M_1 + M_2.$$

The location of the point of inflection is found from

$$\frac{M_1 + M_2}{L} = \frac{M_2}{L_1} = \frac{M_1}{L_2},$$

or

$$L_1 = \frac{LM_2}{M_1 + M_2} \text{ and } L_2 = \frac{LM_1}{M_1 + M_2}.$$

As $f_1 = \frac{M_1}{R}$, we find the angle ω from

$$\sin \omega = \frac{f_1}{L_2} = \frac{M_1 + M_2}{LR}.$$

We can also, as in Fig. 38 c, and without disturbing the equilibrium, add at each of the points A and C two forces, each equal to R in magnitude, but acting in opposite directions. If now one force R is resolved in two components, one parallel to the chord of the deformed bar, and the other at right angles to it, we obtain the same result as before.

In order to test the accuracy of our calculation, we apply now Müller-Breslau's method as a check.

As $\Sigma M = 0$ with respect to any panelpoint, and on account of symmetry we must have in reference to Fig. 39, $M_1 = M_6$ and

$M_2 = M_3 = M_4 = M_5$, consequently there remain only two unknown moments to be determined.

We select M_1 and M_2 as the two unknowns, and begin with the

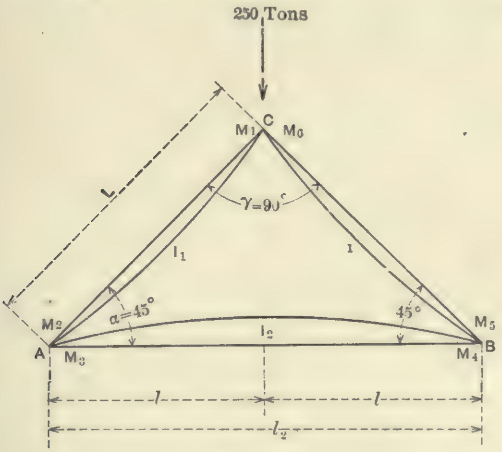


Fig. 39.

calculation of the 6 E fold alterations of the angles in the triangle according to equations (1).

These values are,

$$6\,E\Delta\alpha = 6\,\{(-6-7)\,1 + (-6+6)\,0\} = -78,$$
$$6\,E\Delta\beta = 6\,\{(-6+6)\,0 + (-6+7)\,1\} = -78,$$
$$6\,E\Delta\gamma = 6\,\{(7+6)\,1 + (7+6)\,1\} = +156.$$

The stresses are given in net tons per square inch, Table I, consequently the modulus of elasticity E must be reduced to the net ton and the square inch as the units, and it is assumed to be 14,500. $\Delta\alpha$, $\Delta\beta$, and $\Delta\gamma$ are measured on the arc for a radius $= 1$.

If, for instance, we suppose that the bars AC and BC are non-elastic, but the bar AB elastic, then, according to Chapter II,

$$\Delta\gamma = \frac{6\,\{7(1+1)\}}{6 \times 14500} = 0.000965 \text{ inches};$$

and if measured for a radius equal to the height of the truss, Fig. 40, we have

$$h\Delta\gamma = 282.84 \times 0.000965 = 0.273 \text{ inches.}$$

The value 0.273 inches is the elongation in the bar AB, for the elongation

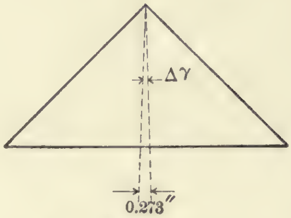


Fig. 40.

$$\Delta\, 2\,l = \frac{S\, 2\, l}{EA} = \frac{125 \times 2 \times 282.84}{14500 \times 17.86} = 0.273 \text{ inches.}$$

Considering the identity of the moments at the supports and apex, we write:

$$U_2 \frac{I_1}{L} = U_3 \frac{I_2}{l_2} = U_4 \frac{I_2}{l_2} = U_5 \frac{I_1}{L};$$

and using the data from Table I, we get

$$U_2 = U_5 \text{ and } U_3 = U_4 = 1.885\, U_2.$$

With reference to equations (31) and (32), we have

$$\left.\begin{aligned} 1.885\, U_2 &= -U_1 - 5.770\, U_2 - 78, \\ U_2 &= U_1 + U_2 - 1.885\, U_2 - 78, \end{aligned}\right\}$$

Solving for U_1 and U_2, we find

$$U_1 = +47.18, \text{ and } U_2 = -16.35;$$

and the bending moments

$$M_1 = U_1 \frac{I_1}{L} = +94.36 \text{ inch-tons, or } 188{,}720 \text{ inch-pounds.}$$

$$M_2 = U_2 \frac{I_1}{L} = -32.70 \text{ inch-tons, or } 65{,}400 \text{ inch-pounds.}$$

Comparing these results with those previously obtained, we find the greatest difference not more than three tenths of one per cent. The finding of a positive M_1 means that the bar AC is deformed at the point C as shown in Fig. 39, but M_2 being found negative, the deformation of the bar at the support is contrary to that shown, and as

$$M_3 = U_3 \frac{I_2}{l_2} = U_2 \frac{I_1}{L} = - 16.35 \times 2 = - 32.70 \text{ inch-tons},$$

the elastic line of the bar AB is also curved contrarily to what is shown in Fig. 39. The deformations are indicated in Fig. 41.

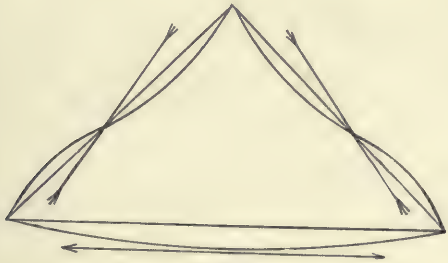


Fig. 41.

The displacements of the lines of direct stresses are,

$$f_1 = \frac{M_1}{S_1} = \frac{94.36}{176.45} = 0.534 \text{ inches.}$$

$$f_2 = \frac{M_2}{S_1} = \frac{32.70}{176.45} = 0.185 \text{ inches.}$$

$$f_3 = \frac{M_3}{S_2} = \frac{32.70}{124.55} = 0.262 \text{ inches.}$$

The foregoing investigation shows that the lines of the displaced stresses form a diagram which is not identical with the truss diagram, consisting of the center lines of the truss members. Equilibrium exists, but the lines of the stresses do not intersect any more in the panelpoints of the truss.

CHAPTER VIII.

OTHER CAUSES OF SECONDARY STRESSES THAN RIVETED JOINTS IN MAIN TRUSSES.

1. Eccentricities.

WHEN the stress in a truss member does not pass through the center of a panelpoint, then its eccentricity not only brings about a change in the secondary stresses, but it itself is a source of a new stress. These stresses have usually different signs, which means that a stress due to an eccentricity, is in part counterbalanced by the rigid connection.

In the execution of calculations, proper attention must be paid to the character of the signs of the moments due to an eccentric connection. So, for instance, we may call a moment positive if it turns in the direction of the hand of a clock, and negative for a counter-revolution.

If a bar with the stress S has an eccentricity at each end c and c_1, then $M = Sc$ and $M_1 = Sc_1$, and the angle of deflection τ_c on account of the eccentricity is, according to equations (2),

$$\tau_c = \frac{l\,\{2\,M - M_1\}}{6\,EI} = \frac{lS\,\{2\,c - c_1\}}{6\,EI}.$$

The angles between the different bars of a truss are now subjected to a change, not only owing to alterations in their lengths, but also on account of eccentricities; and after having taken care of the deflection angles τ_c with their proper signs in the determination of the $E\Delta\alpha$, the calculations proceed then as previously explained. After the calculations are finished, the direct effects of the eccentric connections must be added to the secondary stresses.

2. Loads between Panelpoints in the Plane of the Truss.

These loads are dead and live load and braking forces. The live load between panelpoints could be avoided by designing a bridge with floorbeams and stringers; and in case a floor is riveted to the posts at any point between top and bottom of the main trusses so that the posts are subjected to bending by the braking of trains, the insertion of extra members will transmit the braking forces to the panelpoints without causing bending in the posts.

The calculation of the effects of loads applied between panelpoints is about the same as that for eccentricities. If we have to deal with dead load, for instance, we consider each member individually, calculate the stress s_ω due to its own weight, determine further the deflection angle τ at both ends caused by it, and proceed then similarly as shown for eccentricities. Finally, we add the stresses s_ω and the secondary stresses.

3. Loads between and at the Panelpoints of a Member supposed to turn freely around a Pin.

Under this head comes an eyebar whose secondary stresses are of particular interest, as has been shown by the discussion they caused among engineers. The exact solution of the problem

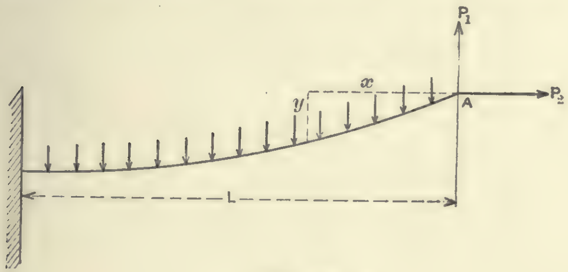


Fig. 42.

requires the simultaneous consideration of the action of its dead weight, which consists in a deflection, and that of a pull reducing in part this deflection. A separate consideration of dead weight and pull leads to approximate results. We assume an eyebar

under tension, the centers of the two heads in a horizontal line and one half of the bar walled in, which circumstance will not in any way disturb the equilibrium. Further, we call P_2 the pull, P_1 the reaction from the dead weight of the bar, supposed to be uniformly distributed over its length, and take A as the origin of coördinates, Fig. 42.

The problem consists now in the representation of the ordinate y as a function of x. The relation between these two variables is the equation of the elastic line; and as soon as the latter is found, it is easy to calculate the deflection, the moment, and the stress for any section of the bar.

With respect to Fig. 42 the equation of the elastic line is written,

$$EI\frac{d^2y}{dx^2} = -P_1x + \frac{P_1x^2}{2L} + P_2y,$$

since the negative ordinates are below the axis of the abscissæ.

Calling the constants $\dfrac{P_1}{EI} = n$ and $\dfrac{P_2}{EI} = q$,

we have
$$\frac{d^2y}{dx^2} = n\left\{-x + \frac{x^2}{2L}\right\} + qy.$$

To facilitate the investigation, we write y' instead of $\dfrac{dy}{dx}$ and y'' instead of $\dfrac{d^2y}{dx^2}$, and our given equation becomes now

$$y'' = qy - nx + \frac{nx^2}{2L}.$$

Differentiating twice, we obtain

$$\frac{dy''}{dx} = qy' - n + \frac{nx}{L},$$

and
$$\frac{d^2y''}{dx^2} = qy'' + \frac{n}{L}.$$

By integrating this last equation twice, we obtain y'' as a function of x; and if this function of x is substituted in the given

equation, we find then at once y as a function of x, which is the object sought.

Multiplying the last equation with dy'', we have

$$dy'' \times \frac{d^2y''}{dx^2} = \frac{dy''}{dx} \times d\left\{\frac{dy''}{dx}\right\} = qy''dy'' + \frac{n}{L}dy''.$$

Since $\frac{dy''}{dx} \times d\left\{\frac{dy''}{dx}\right\}$ is the differential of $\frac{\left\{\frac{dy''}{dx}\right\}^2}{2}$,

therefore $\frac{\left\{\frac{dy''}{dx}\right\}^2}{2}$ is the integral of $\frac{dy''}{dx} \times d\left\{\frac{dy''}{dx}\right\}$,

and we get by integration,

$$\left\{\frac{dy''}{dx}\right\}^2 = 2\int qy''dy'' + 2\frac{n}{L}\int dy'' + A = qy''^2 + \frac{2n}{L}y'' + A,$$

or

$$\frac{dy''}{\sqrt{qy''^2 + \frac{2n}{L}y'' + A}} = dx.$$

Integrating again, we obtain

$$\frac{1}{\sqrt{q}}\, l\left\{\frac{n}{L\sqrt{q}} + y''\sqrt{q} + \sqrt{qy''^2 + 2\frac{n}{L}y'' + A}\right\} + B = x.$$

A and B are arbitrary constants; and as these can be changed, we write the last equation,

$$l\left[\frac{\frac{n}{L\sqrt{q}} + y''\sqrt{q} + \sqrt{qy''^2 + \frac{2n}{L}y'' + A}}{B}\right] = x\sqrt{q},$$

or

$$\frac{\frac{n}{L\sqrt{q}} + y''\sqrt{q} + \sqrt{qy''^2 + \frac{2n}{L}y'' + A}}{B} = \varepsilon^{x\sqrt{q}}.$$

ε is the base of the system of natural logarithms. From this equation we obtain the value of y'' as a function of x, and by substitution of this value in the given equation $y'' = qy - nx + \frac{nx^2}{2L}$, we get after some simple transformations,

$$y = \frac{B\varepsilon^{x\sqrt{q}}}{2q\sqrt{q}} + \left\{ \frac{n^2}{2L^2q^2\sqrt{q} \times B} - \frac{A}{2q\sqrt{q} \times B} \right\} \varepsilon^{-x\sqrt{q}} - \frac{n}{Lq^2} + \frac{nx}{q} - \frac{nx^2}{2Lq}.$$

The next step consists in the determination of the constants. Putting

$$-\frac{n^2}{2L^2q^2\sqrt{q} \times B} + \frac{A}{2q\sqrt{q} \times B} = C, \text{ and } \frac{B}{2q\sqrt{q}} = D,$$

C and D being two new constants, we have

$$y = -\frac{n}{Lq^2} + \frac{nx}{q} - \frac{nx^2}{2Lq} + D\varepsilon^{x\sqrt{q}} - C\varepsilon^{-x\sqrt{q}};$$

and by differentiation,

$$\frac{dy}{dx} = \frac{n}{q} - \frac{nx}{Lq} + \sqrt{q} \times D\varepsilon^{x\sqrt{q}} + \sqrt{q} \times C\varepsilon^{-x\sqrt{q}}.$$

From the conditions that $y = 0$ for $x = 0$, and $\frac{dy}{dx} = 0$ for $x = L$, the two constants C and D are found.

They are

$$C = -\frac{n\varepsilon^{L\sqrt{q}}}{Lq^2\left\{\varepsilon^{L\sqrt{q}} + \varepsilon^{-L\sqrt{q}}\right\}},$$

$$D = \frac{n}{Lq^2}\left\{1 - \frac{\varepsilon^{L\sqrt{q}}}{\varepsilon^{L\sqrt{q}} + \varepsilon^{-L\sqrt{q}}}\right\};$$

and if C and D in the equation for y are replaced by these values, then

$$y = \frac{n}{q}\left\{x - \frac{x^2}{2L} + \frac{1}{Lq}\left(\frac{\varepsilon^{(L-x)\sqrt{q}} + \varepsilon^{-(L-x)\sqrt{q}}}{\varepsilon^{L\sqrt{q}} + \varepsilon^{-L\sqrt{q}}} - 1\right)\right\}.$$

EXAMPLE. An eyebar 15 inches wide by 2 inches thick and 55 feet long is subjected to a pull $P_2 = 600{,}000$ pounds, or 20,000 pounds per square inch. Required the maximum bending stress.

Let the modulus of elasticity be 29,000,000 pounds per square inch,

$$P_1 = 2800 \text{ pounds,}$$
$$I = 562.5,$$
$$L = 330 \text{ inches,}$$
$$n = \frac{P_1}{EI} \text{ and } q = \frac{P_2}{EI}.$$

The maximum deflection D is equal to

$$D = \frac{n}{q}\left\{\frac{L}{2} + \frac{1}{Lq}\left(\frac{2}{\varepsilon^{L\sqrt{q}} + \varepsilon^{-L\sqrt{q}}} - 1\right)\right\} = 0.488 \text{ inches.}$$

The maximum bending moment is equal to

$$M = -2800 \times 330 + 2800 \times 165 + 0.488 \times 600{,}000,$$

or $M = -169{,}200$ inch-pounds, consequently the maximum bending stress equals $\frac{169{,}200}{75} = 2256$ pounds per square inch.

This bending stress amounts to about 11.3 per cent of the direct stress, and is compression in the top fiber and tension in the bottom fiber.

4. Changes in Temperature.

Any rise or fall in temperature affects the lengths of truss members. These alterations in the lengths of bars produce deformations of a truss, which may or may not be connected with stresses.

A statically determinate truss whose movable end is free from any frictional resistances and whose members can turn freely around pins, is not subjected to any temperature stresses, not even when its members are unequally affected by temperature.

But the case is different with statically indeterminate trusses, no matter whether the indeterminateness refers to the outer or

inner forces. The safety of the structure requires an examination of its temperature stresses, and, if necessary, an inquiry into the secondary stresses arising from them. Statically determinate trusses, which are riveted, can also be affected by temperature stresses. So, for instance, can a top chord have a higher temperature than a bottom chord, if the latter is protected from the rays of the sun by a floor. In this case the difference in temperature produces an upward deflection of the truss, and consequently deformations and stresses in the bars.

The calculation of temperature stresses is as follows:

If c is the coefficient for expansion or contraction due to 1° change in temperature, and t the total change of temperature in degrees, then

$$\frac{Sl}{EA} = ctl,$$

or

$$\frac{S}{A} = s_t = cEt.$$

This value of s_t is now used, — for instance, in equations (1), — in order to calculate the angle alterations, whereupon the succeeding operations are executed as formerly explained.

5. Misfits.

It is very essential that the lengths of members for a riveted truss are exact; if they are not, they are a source of secondary stresses. We speak here in particular of statically indeterminate trusses where a small shop mistake may lead to a considerable change in the stresses. In order to determine the effects of these misfits on the stresses, we assume that these misfits are produced by stresses s_m, and if we call Δl the amount a truss member is either too long or too short, we can write

$$\frac{s_m l}{E} = \Delta l,$$

or

$$s_m = \frac{\Delta l}{l} \times E.$$

This unit stress s_m is now used in the same way as the unit stress s_t due to a change in temperature.

6. Brackets on Posts.

Eccentric loads on posts caused by brackets give rise to very lengthy calculations, and as these loads affect the entire cross-frame of a bridge we cannot consider them here. It is best to avoid such brackets wherever possible.

7. Unsymmetrical Connections.

Good practice does not allow unsymmetrical connections in the design of main trusses; and at such places where they are tolerated, the secondary stresses caused by them are of minor importance.

8. Curved Members.

The secondary stresses due to curved members can be computed from the suggestions given for eccentric connections. Bridge trusses with curved members do exist, but, in the opinion of the writer, they are utterly out of their proper place. The defense of such members, even from an æsthetic point of view, is weak.

9. Pin Joints.

We believe it to be a safe statement that many an engineer lays too great a stress on the value of a pin joint as a means to reduce secondary stresses. Of course no pin joint is perfect, and in some cases the frictional resistance may be so small that we do not need to pay any attention to it. On the other hand, if a pin is designed with no consideration whatever for a reduction of secondary stresses, there is at least a chance that it will be ineffective, so that a riveted joint could just as well have been built.

If a bar does turn around a pin, it is certain that the stress in the bar will be displaced out of its former axial position, Fig. 43. In this case the displacement r is such that the moment Sr over-

comes the frictional moment $F \times R$, F being the frictional resistance and R the radius of the pin.

The frictional resistance is found by resolving the stress S at its point of application on the pin periphery into two components, the line of one component coinciding with the tangent on the pin periphery, and the other normal to this tangent. The frictional

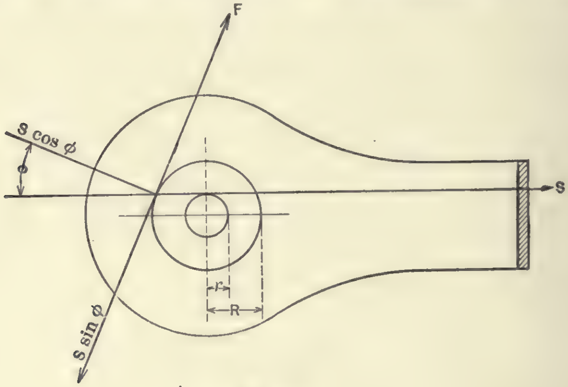


Fig. 43.

resistance is now equal to the tangential stress $S \times \sin \phi$ and also equal to the normal pressure $S \times \cos \phi$ multiplied by the coefficient of friction. The angle ϕ included between the line of the stress S and the normal is the angle of friction.

We have then,

$$F \times R = S \times \sin \phi \times R = S \times r,$$

and $$r = R \times \sin \phi.$$

This shows the displacement r is independent of the stress S. If this displacement is smaller than r, then no turning of the bar around the pin is possible; and if it equals r, a turning takes place.

Let us now go back to the triangular riveted truss, calculated in the preceding chapter, in order to find for a given coefficient of friction the greatest diameter of the pin at the apex, which must not be exceeded if a turning of the bars around the pin is to be realized. As the displacement of the stress in this case was

found $= 0.534$ inches, and assuming 0.2 as the value of the coefficient of friction, we have $0.534 = 0.2\ R$, or $R = 2.67$ inches, which means that if we design this pin with a greater diameter than $2 \times 2.67 = 5.34$ inches, it would be ineffective, and with a greater frictional coefficient the diameter of the pin should of course be still smaller.

From the above considerations it follows that if we wish to reduce the secondary stresses to a minimum in hinged members, it is very essential to keep the size of the pins down as much as the strength and safety of a structure will permit.

The writer is not aware that experiments have been made with a view to determine the frictional coefficient for cases that are here under consideration, and therefore he is not in a position to furnish any reliable data.

Some specifications require that the diameter of a pin shall not be less than three quarters of the width of any eyebar which they connect. Calling S the direct stress of an eyebar, w and t its width and thickness, 0.2 the frictional coefficient, then we have for the displacement of the stress, $r = 0.375\ w \times 0.2 = 0.075\ w$. The direct unit stress equals $\frac{S}{wt}$, and the bending stress is equal to the moment divided by the section modulus, or equal to

$$\frac{S \times 0.075 \times w \times 6}{w^2 t} = \frac{S}{wt} \times 0.45.$$

The sum of the direct and bending stress equals $\frac{S}{wt} \times \{1 + 0.45\}$. which means that the secondary stress amounts to 45 per cent of the primary stress.

In regard to these stresses it should be noted that probably the vibrations due to a passing train cause the eyebars to adjust themselves to their original positions; they probably turn around their pins, whereby the angle of friction is momentarily decreased from what it would be for static loads.

10. Friction at Supports.

The frictional resistances at the supports of trusses are dependent on the coefficient of friction, the vertical loads, and the length of span. These resistances would be very great indeed for long trusses, if sliding friction were allowed; but as for such trusses only rolling friction is allowed, the resistances are very considerably reduced, and need not to be considered. But it is, of course, important that the roller ends be kept in proper working order.

11. Cross-Frames.

The analytical discussion of cross-frames, if a complete solution of the problem is attempted, leads to very extensive investigations and is outside of our province. Nevertheless, we will give a few remarks.

A thorough examination of cross-frames for either a through or deck railway bridge consisting of verticals, floorbeams, and cross-constructions of any description, would consider the bending effects on the frame of the following forces: dead load of floorbeams, dead and live loads transferred from the stringers to the floorbeam, centrifugal force, impact, wind pressure against the train, wind pressure against the structure concentrated at top and bottom and uniformly distributed against the posts, unequal deflection of the main trusses, and unequal change of temperature for different parts of the frame.

The influence of these forces is felt in the entire cross-frame, causing also bending and twisting in the members of the main trusses.

The secondary stresses in cross-frames are next in importance to those in the main trusses; and concerning cross-frames with no diagonals, it may be said that they are predisposed to higher stresses.

For the purpose of finding out to what extent the secondary stresses are affected in the verticals by dead and live load, and exclusively by changing the dimensions of the verticals, the writer

examined a riveted cross-frame of a two-track railway bridge, consisting of a 6-foot-deep floorbeam, a lattice strut, and two suspenders, each of the latter composed of 4–8-inch bulb angles with a total area of 22.5 square inches.

Deep floorbeams with large moments of inertia tend toward a reduction of secondary stresses.

The trusses were 30 feet from center to center, the panel length 27 feet, and the live load carried by each of the suspenders 187,000 pounds.

The lengths of the verticals and their depths parallel to the web of the floorbeam were the only dimensions changed; and if we express the secondary stresses in percentages of the unit stress due to dead + live load, the results are as follows:

	Per cent.
Verticals 36 feet long and 14½ inches deep	13.5
Verticals 36 feet long and 29 inches deep	23.3
Verticals 18 feet long and 14½ inches deep	25.2
Verticals 18 feet long and 29 inches deep	40.9

A cross-frame as described 36 feet deep may belong to a through Warren truss bridge, the verticals being suspenders; and a cross-frame 18 feet deep may belong to a Baltimore deck bridge with the top chord projecting above the floor, the verticals being short posts. For the analytical treatment it does not make any difference whether the floorbeam is at the bottom or top.

In another example, the writer selected a cross-frame of a four-track railway through bridge, with two main trusses, symmetrically loaded, gave purposely the post the unusual transverse depth of 52 inches, designed it under three prominent specifications, and found in each case that the secondary stress amounted to closely 55 per cent of the primary stress. But by reducing the depth parallel to the floorbeam of this comparatively short post to 18 inches with ample provision against buckling, the secondary stress was reduced to 22.3 per cent.

These high stresses in the verticals are verified by Winkler, *Querkonstruktionen*, pp. 179–182; by Jebens, *Die Spannungen in*

den Verticalständern der eisernen Brücken, Zeitschrift des Vereins deutscher Ingenieure, 1880, p. 127; and by that standard work of engineering science, *Handbuch der Ingenieurwissenschaften*, vol. II.

From the above we can draw the conclusion that a small ratio between the length and the transverse depth of a vertical should be avoided, and that the transverse depth should be restricted to a proper limit, otherwise we run the risk of exceeding by far the unit stresses as prescribed by our specifications.

It is of interest to note that investigations of riveted cross-frames disclose the assumption of the fixity of floorbeams at the posts as quite erroneous; the floorbeams cannot even be approximately so considered.

12. Yielding of Foundations and Settlements of Masonry.

The influence on the stresses of a truss caused by yielding of the foundations or settlements of masonry could be calculated if these displacements were known. As a matter of fact, they can only be estimated or judged from uncertain or incomplete evidence.

In regard to these displacements we must distinguish between elastic and non-elastic deformations. Elastic deformations vanish as soon as the load is removed, while the non-elastic are permanent.

A structure should not be built if the computations prove that it is very sensitive to assumed displacements of its supports, provided we cannot give it supports which are almost as good as fixed. It is also essential in such a case that the supports are placed with the utmost care in those positions as assumed in the computations.

Of course not every truss is affected by a displacement of its supports. So, for instance, a truss, resting on two supports and with one movable end, a cantilever truss or a three-hinged arch are free from this influence. On the other hand, a continuous truss is very susceptible to the influence of a yielding of the foundations or a settlement of the piers (particularly so if very massive

and deep); also the one-hinged, the two-hinged, and the hingeless arch and others are affected by these causes.

Continuous truss over three supports. We will suppose that the center pier of a continuous truss over three supports yields vertically to the amount of D inches, Fig. 44. Such a displacement reduces the center reaction an amount X, which, when found, enables us to compute the stresses in the truss members, and consequently also the secondary stresses. In order to find X, we

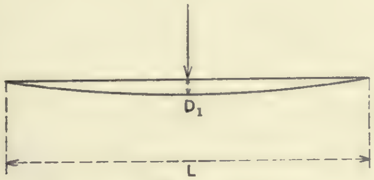


Fig. 44.

place a load equal to unity at the center of the truss, the latter assumed to be resting on its end supports only, and determine the deflection D_1 at the center due to this load = 1. If we now apply a load X at the center of the truss, then the deflection will be XD_1; but as this deflection must be equal to the supposed deflection D, we have

$$XD_1 = D, \text{ or } X = \frac{D}{D_1}.$$

The examination of a continuous plate girder is quite similar to that of a truss.

Two-hinged arch. Of all forms of arches, the two-hinged arch has probably found the widest application, and from this reason we will take it as an example. This kind of arch is statically indeterminate with one unknown quantity, which is the horizontal thrust, provided the truss has no redundant members.

The principle of the derivative of work furnishes us with convenient means to determine the thrust. For this purpose we first remove the statical indeterminateness by giving the truss one movable end, the two supports being supposed in this case in

a horizontal line. The stress S_0 in any of the truss members, due to vertical loading, can now be found by statics. Thereupon we apply a force = 1 at the movable end, acting toward the fixed hinge, and determine the stress v either analytically or graphically in every member of the truss. If now the real horizontal thrust $= H$, and if S denotes the stress in any of the bars of the statically indeterminate truss, we have

$$S = S_0 + vH.$$

The stress S_0 is a function of the exterior forces, and is independent as well as the stress v of the thrust H. The work of deformation W is expressed by

$$W = \sum \frac{S^2 l}{2\,EA},$$

where l = length of any bar,
A = sectional area of any bar,
E = modulus of elasticity.

The principle of the derivative of work states that, if we express the work of deformation of the bars as a function of the exterior forces, then the displacement of the point of application of a force (in our case H) equals the partial derivative of the work of deformation with respect to that force.

Differentiating the work of deformation with respect to H, we get

$$\frac{\partial W}{\partial H} = \sum \frac{Sl}{EA} \times \frac{\partial S}{\partial H} = -\,\Delta L,$$

L being the length of the span from center to center of end pins, and ΔL the supposed horizontal outward yielding of the masonry supports under the action of the vertical loading.

In case temperature stresses are to be considered, we call the alteration in the length of a bar: $\frac{Sl}{EA} + \varepsilon tl$ instead of $\frac{Sl}{EA}$; ε being the coefficient for extension or contraction due to a change in temperature of 1 degree and for a unit of length, and t the total change in temperature.

But $S = S_0 + vH$, and $\frac{\partial S}{\partial H} = v$, consequently we obtain by substitution,

$$\sum \frac{S_0 l}{EA} \times v + H \sum \frac{v^2 l}{EA} + \sum \varepsilon t l \times v = - \Delta L$$

or

$$H = - \frac{L + \sum \frac{S_0 l}{EA} \times v + \sum \varepsilon t l \times v}{\sum \frac{v^2 l}{EA}} .$$

The derivation of this formula for H assumes that no initial stresses exist, that is to say, with the removal of the outer loading all stresses must vanish.

The three different causes which influence the thrust of the arch may also be considered separately.

The thrust caused by the sole action of the vertical load is

$$H = - \frac{\sum \frac{S_0 l}{EA} \times v}{\sum \frac{v^2 l}{EA}},$$

and by a change in temperature,

$$H = \frac{\sum \varepsilon t l \times v}{\sum \frac{v^2 l}{EA}},$$

[t being positive for an increase] and by a change in the length from center to center of end hinges,

$$H = - \frac{\Delta L}{\sum \frac{v^2 l}{EA}}$$

[ΔL being positive for an increase].

The expressions $\sum \frac{S_0 l}{EA} \times v$ and $\sum \frac{v^2 l}{EA}$ designate horizontal displacements of the hinges, which are supposed to move freely; the former is the displacement caused by the vertical load, and the latter is that due to a horizontal thrust equal to unity and applied at the hinge.

The length of the distance between the hinges can be increased by the thrust of the arch in pushing the supports bodily outward or by crushing or compressing the masonry. Such an action naturally decreases the horizontal thrust and consequently exercises an influence on the stresses in every member of the arch truss with corresponding changes in the secondary stresses.

As an example of a thorough examination of the effects of the yielding of masonry supports on the stresses in a truss, can be mentioned the bridge across the Emperor William canal at Gruenenthal, described by Fülscher in "Zeitschrift für Bauwesen," 1898. This bridge is built for a single-track railway and highway traffic, has two crescent-shaped arch ribs, projecting above the floor system, and measures 513.3 feet from center to center of end pins.

The wind bracing in the plane of the floor consists of a wind chord and diagonals, and the place of the struts is taken by the floorbeams. It is divided into three sections: a central section, lying between the intersection points of the arch ribs and the floor; and two end sections, each extending from these intersection points to the abutments. The central section of the bracing transfers the wind pressure to the arch, and the end sections partly to the arch ribs and partly to the abutments.

In the central section the floor is suspended, and in the end sections it is supported by posts resting on the arch ribs.

Apart from the effects of the dead load, live load, wind pressure, and temperature changes, the stresses in each truss member have been calculated under the supposition that, before riveting up the wind chords, but after the arch carried its own weight, a horizontal yielding of each of the masonry supports of $1\frac{3}{16}$ of an inch would take place, and a further yielding of the supports of $\frac{5}{16}$ of an inch under the influence of the live load.

While the two-hinged arch is affected only by horizontal displacements of the masonry, the one-hinged and hingeless arch are susceptible to horizontal and vertical displacements, and, moreover, to a possible turning of the masonry in the plane of the truss.

CHAPTER IX.

IMPACT.

It is outside the scope of this book to take the reader over the field of mathematical investigations of dynamical effects on bridges, or to discuss the many suggestions that have been made, how impact and vibrations could be covered in our specifications. If it had been the intention, it would have been proper to extend the discussion of secondary stresses under static loads to the cross-sections of bridges, floors, and wind bracings first, before giving some notes on the secondary stresses under moving loads. But a few words on this subject are not out of place.

While the progress in the theory of bridges has been gigantic, the same cannot be said of the theory of dynamical effects on bridges, and the reason for this is not far to seek.

If every element had to be considered which has some connection with the effect produced on a bridge by a fast-moving train, then the problem of impact would, of course, be insoluble, and even in the simplest case. But barring such elements, as, for instance, a defective track, or inequalities of the rail ends at rail splices, etc., whose influence is naturally outside the province of a calculation, the mathematical difficulties presented by the problem are almost unsurmountable; and, in fact, they have been only overcome for the case of a single load moving over a beam.

A train in passing over a bridge causes the latter to deflect, whereby the pressure or centrifugal force exerted by the train against the bridge is influenced by the deflection and the velocity of the moving masses, and this pressure in turn exercises an influence on the form of the deflection curve. The mutual relations between the quantities which enter into consideration are compli-

cated, and made more so by the counterweights of the locomotive drivers, which affect the values of the pressures. In consequence of the great velocity with which a train enters a bridge, of the variable loads produced by the counterweights of the locomotive, of defective rail splices and unround wheels, the bridge is subjected to vibrations. Not only does the bridge as a whole vibrate vertically and horizontally, but also the different bars perform rapid oscillations longitudinally and transversely.

The view held by some engineers, that a fast-moving train does not give a truss the necessary time to offer its full resisting power, does not harmonize with the fact that the propagation of stresses in elastic bodies follows the laws governing the velocity of sound. The velocities of sound vary greatly in different mediums; in liquids the velocity is greater than in air, and in solids the range is rather wide. In caoutchouc the velocity is from 100 feet to 200 feet per second, while in steel wire, wrought iron, and steel it amounts in round figures to 16,000 feet, or about 3 miles per second. A telegraph wire furnishes a good illustration of the propagation of sound in solids. Filing at one end of the wire can be heard at a distance of several miles by placing the other end in the ear.

The subject in question has been treated from various points of view. In the year 1890, Professor Ritter, of Aachen, published calculations as a result of theoretical considerations which place the velocity of the propagation of impulses in wrought iron as high as three miles per second, and Professor Mach showed optically the propagation of longitudinal vibrations. Professor Radinger treats the subject in his book on steam engines with high piston velocities, published in Vienna, 1892. In the same year appeared in "Zeitschrift des österreichischen Ingenieur- und Architekten-Vereins," a paper of great interest to engineers on "Metal Constructions of the Future" by the late Professor Steiner.

In this paper Steiner shows how impulses can be made visible to the eye by the construction of a model of a bridge truss, each bar of the truss to be provided with a groove. If such a model is placed in a horizontal position and the grooves filled with quick-

silver, it is then possible to follow the propagation of an impulse, which has been made by the finger at any point of the truss.

However, it should be remarked that the propagation of stresses from section to section in the members of a bridge truss experiences a delay, firstly, because the line of progress must be changed, and secondly, on account of the imperfections of the joints. A pin-connected truss appears to be at a disadvantage compared with a riveted steel truss, as the latter resembles more closely a continuous mass. But, whatever may be left of the velocity of propagation of stresses, it appears to be of sufficient magnitude to be looked upon as instantaneous.

Under the assumption that the stresses, in traversing a truss, encounter so many difficulties which reduce their velocity, Radinger arrives at the conclusion that a truss may be subjected at the ends to particular high stresses by fast-running trains. He arrives at this conclusion in this way: Let us suppose for an illustration that the velocity of stresses is reduced from 16,000 feet per second to 4000 feet per second, then the time required by a span of 400 feet in length to act as a structure on two supports would be $2 \times \frac{400}{4000} = \frac{1}{5}$ of a second. A train running over the bridge with a speed of 90 feet per second would cover a length of 18 feet in $\frac{1}{5}$ of a second, which means that for this time the bridge would have only one support for the live load.

The solution of the problem of impact, but only in the simplest case, namely for a single constant load, moving over a weightless beam of uniform sectional area and resting on two rigid supports, has been rigidly effected by Dr. H. Zimmermann, whose brilliant researches are contained in his paper, "Die Schwingungen Eines Trägers Mit Bewegter Last," Berlin, 1896. Many investigators have attacked the problem without success on account of the mathematical difficulties. But in this respect it should be remarked that the general integral of the differential equation of the curve described by the moving mass is of a kind that was unknown up to the publication of Zimmermann's writing, although he knew it

as early as 1892. Zimmermann tells us that he hesitated to publish the purely mathematical solution of the problem, as he had wished to work out a practical method of calculation, but the great amount of time and labor spent on this undertaking caused a considerable delay of his publication.

Zimmermann's extensive paper is naturally of a highly mathematical character, and his penetration into the subject is deep. Therefore, we will give only the results of his investigations, which may be applied in practice.

As has been said before, a weightless beam of uniform sectional

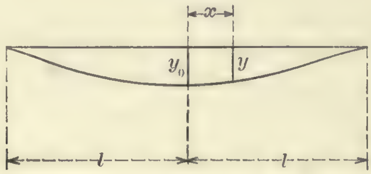


Fig. 58.

area and resting on two rigid supports is assumed, over which moves a mass with any constant velocity. If the mass moves over the beam so slowly that the pressure against the beam is invariable, we have the familiar equation,

$$y = y_0 \left\{ 1 - \frac{x}{l} \right\}^2 \left\{ 1 + \frac{x}{l} \right\}^2,$$

which expresses the form of the deflection curve, Fig. 58.

The object is now to find for greater velocities the path described by the moving mass, in which case the pressure against the beam is variable. In other words, if the curve shall be found, it is necessary to consider the effect of the centrifugal force of the moving mass. The differential equation expressing the desired curve is

$$\frac{d^2\eta}{d\xi^2} + \alpha \frac{\eta}{\left\{1 - \xi^2\right\}^2} - \beta = 0,$$

and the meaning of the symbols is as follows: The time t_0 required by the mass to cover one half of the span length l with the velocity c is $t_0 = \frac{l}{c}$; and if we suppose the mass were to descend from the height h inside the same time t_0, then $2h = gt_0^2 = g\frac{l^2}{c^2}$, g being the acceleration of gravity.

The ratio

$$\frac{2h}{y_0} = \frac{g\frac{l^2}{c^2}}{\frac{Wl^3}{6EI}} = \alpha, \text{ and } \frac{P}{W} = \beta.$$

E and I are modulus of elasticity and moment of inertia of the beam, and P equals the weight W of the mass m + the load W_1 transferred to the wheel by the spring.

Zimmermann puts further $\frac{x}{l} = \xi$, and takes the fall $2h$ as a unit of measure for y by writing $\eta = \frac{y}{2h}$.

α and β are constants, if P and c are assumed to be invariable, but Zimmermann's method of integration can also be employed in case β is variable, representing an arbitrary function of ξ.

The form of the path described by the moving mass is dependent on α in a high degree. This curve is known for static loads where $\alpha = \infty$ for $c = 0$. The total pressure P does not influence α. If P changes, then the ordinates of deflection for both the moving mass and the static load change also, but in the same proportion. The stresses of the beam are naturally greatest for the maximal values of W and P.

In order to find a value for α, which can be used in practice, we determine first two limiting values. If there are no springs assumed, we put $W = P$, and

$$\alpha = \frac{g\frac{l^2}{c^2}}{\frac{Pl^3}{6EI}}.$$

The moment M_c at the center of the beam is $M_c = \frac{Pl}{2}$; and if s is the stress at the extreme fiber, and D the depth of the beam, we have

$$\frac{Pl}{2} = \frac{2\,Is}{D},$$

and

$$\alpha = \frac{3\,gDE}{2\,c^2 s}.$$

If we put $W = \frac{1}{4}P$ for a perfect spring, then

$$\alpha = \frac{6\,gDE}{c^2 s}.$$

As the wheel load is neither rigidly supported nor a spring perfect, we will assume

$$\alpha = \frac{3\,gDE}{c^2 s}.$$

Zimmermann has conclusively shown that for velocities up to 62 miles per hour, or 91 feet per second, the path of the moving mass can be considered as symmetrical about the center line of the beam, so that the stress of the beam can be determined from the deflection at the center by the sufficiently accurate equation

$$Y = 1 + \frac{4}{\alpha - 12}.$$

Consequently the greatest proportionate increase of the deflection, or of the bending moment, or of the stress, is expressed by

$$\text{Increase} = \frac{1}{\frac{1}{4}\alpha - 3}.$$

In the equation for α, the quantities g, D, and c are expressed in feet, and E and s in tons per square inch. α is independent of the length of the span, nevertheless the equations can only be used for

very short spans on account of the assumption that only one load moves over the beam. Assuming, for example,

$$\begin{aligned} D &= 1 \text{ foot}, \\ c &= 91 \text{ feet per second}, \\ s &= 8 \text{ tons per square inch}, \\ E &= 14{,}500 \text{ tons per square inch}, \end{aligned}$$

we find

$$\alpha = \frac{3 \times 32.17 \times 1 \times 14500}{91^2 \times 8} = 21,$$

which gives about 44 per cent impact. For

$$\begin{aligned} D &= 1.5 \text{ foot}, \\ c &= 91 \text{ feet per second}, \\ s &= 4 \text{ tons per square inch}, \\ E &= 14{,}500 \text{ tons per square inch}, \end{aligned}$$

$$\alpha = \frac{3 \times 32.17 \times 1.5 \times 14500}{91^2 \times 4} = 63,$$

which reduces the impact to about 8 per cent.

A further result of Zimmermann's investigation can be summed up in the statement that the end of the girder and its support in the direction of the motion of the load is subjected to a particular impact, which increases in intensity with the rigidity of the support, the rails, and the tires.

CHAPTER X.

EXAMPLES AND CONCLUDING REMARKS.

A FEW years ago Professor E. Patton in Kiew, Russia, published a book on secondary stresses in the Russian language, whose title, translated into English is, "Calculation of Trusses with Stiff Joints," Moscow, 1901. This book contains a number of examples of bridge trusses, calculated by German authors, which Professor Patton collected from various books and periodicals, besides enriching this collection by a number of examples of his own. An abstract of this book is published in "Zeitschrift für Architektur und Ingenieurwesen," Hannover, 1902. Through the courtesy of Professor Patton the writer is enabled to republish some examples of the above collection, adding three trusses of American design.

It is hardly necessary to remark that although a truss is of foreign origin it cannot fail to be serviceable in the study of secondary stresses.

The calculation of secondary stresses is naturally always preceded by that of primary stresses, which is executed under the assumption that the bars can turn around frictionless pins and in this case only do the lines of stresses coincide with the axes of the bars. But this assumption is never fulfilled in our trusses. Either we have a riveted joint or a pin joint with frictional resistances of more or less severity.

As we have seen in previous discussions it is owing to the nature of joints that the bars become deformed under the action of loads upon the truss and the lines of stresses displaced, and moreover, the primary stresses in a riveted truss for a given load are not identical with those found under the assumption of frictionless pins. But the differences in the primary stresses between a

riveted and a pin-connected truss, all other conditions being equal, is inconsiderable and consequently can be neglected for all practical purposes.

We have also explained that the calculation of secondary stresses does not need to take into consideration the deformations of the bars, provided sufficient provision against buckling has been made. In this case the stresses in any given section of a bar are dependent only on the bending moment with respect to that section, and these bending moments generally reach their greatest values at the ends of the bars. After the bending moments have been found we can then calculate two different stresses in the extreme fibers at each end of each bar according to known formulas.

Intersection points of diagonals, which are riveted at these points, are treated in exactly the same manner as any other panel points.

The designation of the letters in the tables is as follows:

I is the moment of inertia of a bar,

l is the length of a bar,

b is the width of a bar,

e is the distance from the neutral axis of a bar to the extreme fiber.

The primary stresses are given per unit of area, and the secondary stresses, which are tension on one side of the bars and compression on the other side, are expressed in percentages of the primary stresses. The maximum total stresses are of course obtained by adding primary and secondary stresses of the same signs.

A study of secondary stresses proves to be instructive, for it discloses the weaknesses in our designs, but at the same time we are also taught how to minimize the defects resulting in stronger and therefore safer bridges.

Besides the character of the truss, which plays an important rôle in regard to the magnitude of secondary stresses, we must also point out the marked influence on these stresses exercised

TABLE 2 AND FIG. 45.

Span 15 m. Wrought Iron.

Member.		Gross area, sq. cm.	I gross $cm.^4$	I cm.	b = 2e cm.	Most danger-ous fiber.	$e_1 = e_2$ cm.	Stresses. Kg./sq. cm	Stresses. %	$\frac{l}{e}$	Sections in mm.
Top Chord	2–4	60	1000	500	19	Top	9.5	−567	14	53	 90 x 90 x 9
	4–6	60	1000	500	19	Top	9.5	−467	18	53	
Bottom Chord	1–3	50	750	500	17	Bottom	8.5	+360	28	59	 80 x 80 x 9
	5–7	50	750	500	17	Bottom	8.5	+300	21	59	
	3–5	70	1125	500	17	Bottom	8.5	−457	18	59	 80 x 80 x 12
Diagonals										$\frac{l}{b}$	
	1–2	50	790	350	17	Top	8.5	−520	6	21	 80 x 80 x 9
	6–7	50	790	350	17	Top	8.5	−420	4	21	
	3–4	30	440	350	17	...	8.5	+100	65	21	 80 x 40 x 7
	4–5	30	440	350	17	...	8.5	−200	34	21	
	2–3	40	610	350	16	...	8	+575	12	22	 75 x 75 x 8
	5–6	40	610	350	16	...	8	+450	19	22	

Secondary stresses are calculated for gross moments of inertia.

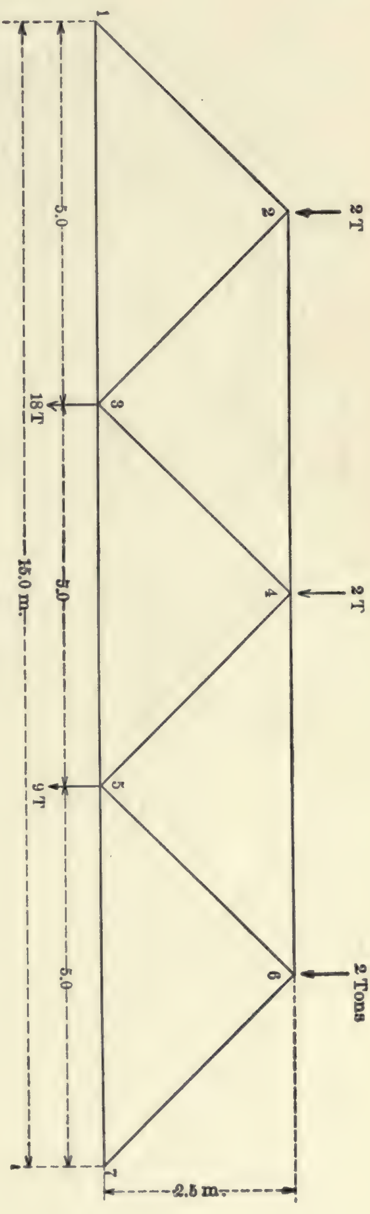


Fig. 45.

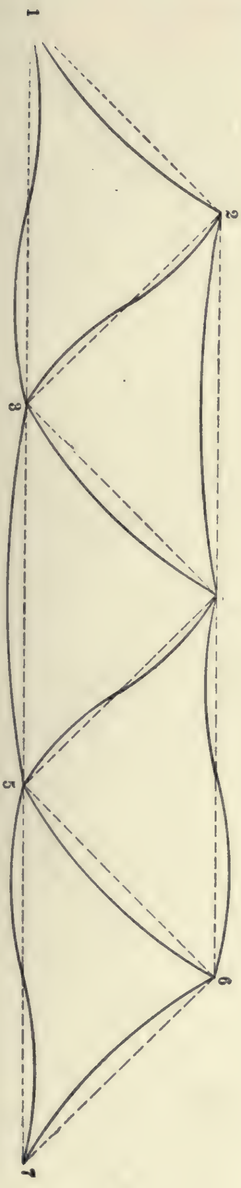


Fig. 45 A.

Warren Bridge Truss. Calculated by Professor Mohr and taken from his paper in "Civilingenieur," 1892.
Secondary stresses determined for positions of loads as indicated.

TABLE 3 AND FIG. 46.

Span 20 m. Wrought Iron.

Member.		Gross area, sq. cm.	I gross cm.^{4}	l cm.	b = 2e cm.	Most dangerous fiber.	$e_1 = e_2$ cm.	Stresses. Kg./sq. cm.	Stresses. %	$\frac{l}{e}$	Sections.
Top chord	1–3	65	4000	400	24	Top	12	−490	22	33	Symmetrical
	3–5	65	4000	400	24	Top	12	−740	12	33	
Bottom chord	0–2	44	400	400	13	Bottom	6.5	+360	25	62	Symmetrical
	2–4	60	800	400	16	Bottom	8	+670	12	50	
	4–4′	60	800	400	16	Bottom	8	+800	7	50	
Diagonals										$\frac{l}{b}$	
	0–1	60	900	360	17	...	8.5	−480	10	21	Symmetrical
	2–3	44	400	360	13	...	6.5	−320	21	28	
	4–5	44	900	360	13	...	6.5	0	...	28	
	1–2	40	300	360	10	...	5	+730	5	36	Symmetrical
	3–4	44	400	360	13	...	6.5	+320	9	28	

Secondary stresses are calculated for gross moments of inertia.

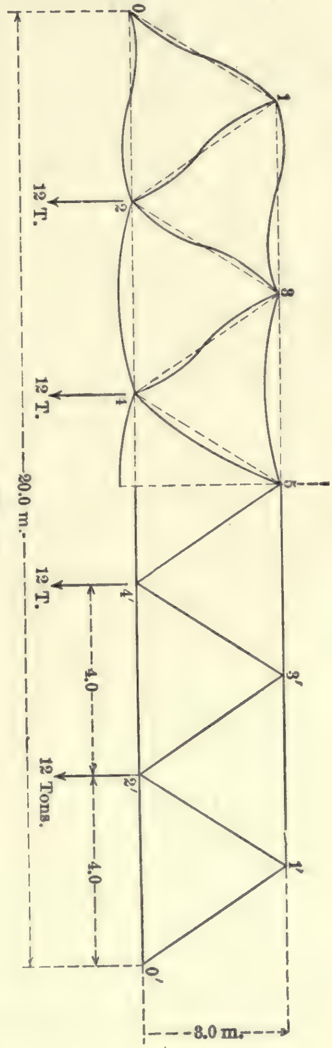


Fig. 46.

Warren Bridge Truss. Calculated by Professor Müller-Breslau and taken from his paper in "Zeitschrift les Architekten-und Ingenicur-Vereins zu Hannover," 1886.

Secondary stresses determined for positions of loads as indicated.

TABLE 4 AND FIG. 47.

Span 36 m. Wrought Iron.

Member.		Gross area, sq. cm.	I gross cm.[4]	l cm.	b = 2e cm.	Most dangerous fiber.	$e_1 = e_2$ cm.	Stresses. $\frac{\text{Kg.}}{\text{sq. cm.}}$	Stresses. %	$\frac{l}{e}$	Sections.
Top Chord	0–2	86	2040	400	27.2	Top	13.6	−410	66	29	
	2–4	166	7110	400	36	Top	18	−560	34	22	
	4–6	226	13000	400	40	Top	20	−600	18	20	
	6–8	253	13000	400	40	Top	20	−630	22	20	
	8–8′	255	15100	400	40	Top	20	−660	18	20	Symmetrical and star shaped, made from flats and angles.
Bottom Chord	A–1	48	2140	200	27.6	Bottom	13.8	0	...	14	
	1–3	95	4270	400	27.6	Bottom	13.8	+700	23	29	
	3–5	182	5960	400	32	Bottom	16	+640	24	25	
	5–7	224	12260	400	40	Bottom	20	+670	19	20	
	7–9	239	13000	400	40	Bottom	20	+690	15	20	
Diagonals										$\frac{l}{b}$	
	0–1	96	12800	429	40	...	20	+750	9	11	
	2–3	77	6550	429	32	...	16	+700	16	13	Symmetrical, made from flats.
	4–5	56	3660	429	28	...	14	+650	20	15	
	6–7	40	740	429	19.2	...	9.6	+460	28	22	Symmetrical, made from angles.
	8–9	48	1130	429	22.2	...	11.1	+14	429	19	
	1–2	136	6920	429	36	...	18	−520	35	12	
	3–4	117	5690	429	35.6	...	17.8	−450	38	12	Symmetrical and star shaped, made from flats and angles.
	5–6	89	3200	429	29.6	...	14.8	−390	43	14	
	7–8	55	1640	429	25.2	...	12.6	−310	32	17	
End Vertical	A–0	143	4410	380	25.2	...	12.6	−500	44	15	

Secondary stresses are calculated for gross moments of inertia.

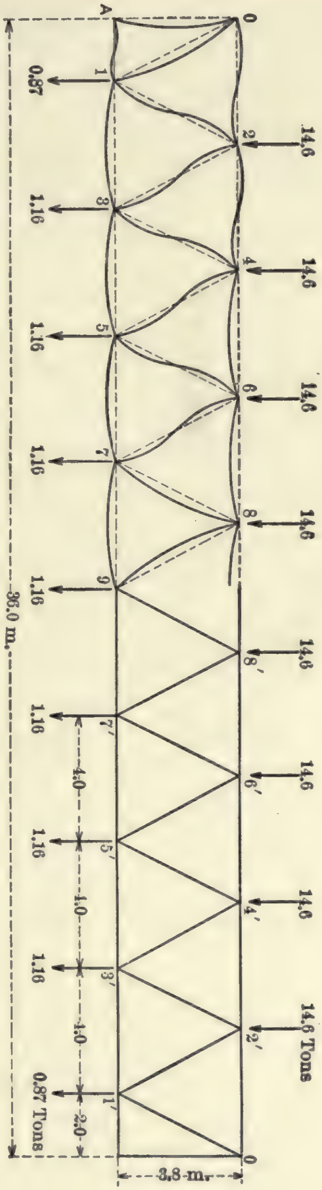


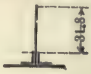
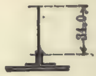
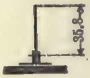
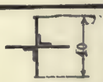
Fig. 47.

Deck Warren Truss of a Railroad Bridge. Calculated by Professor Manderla and taken from his paper in "Allgemeine Bauzeitung," 1880.

Secondary stresses determined for positions of loads as indicated.

TABLE 5 AND FIG. 48.

Span 42 m. Wrought Iron.

	Member.	Gross area, sq. cm.	I gross cm.[4]	l cm.	b cm.	Most danger-ous fiber.	e cm.	Stresses. Kg./sq. cm.	Stresses. %	l/e	Sections in cm.
Bottom Chord	0 –1	106	...	350	41	Bottom	9.0	+280	54	39	 1 Web — 40 x 1 1 C. Pl.— 40 x 0.8 2 ∟s — 9 x 9 x 1
	0′–1′	106	...	350	41	Bottom	9.0	+160	19	39	
	1 –3	106	...	350	41	Top	31.8	+280	143	11	
	1′–3′	106	...	350	41	Bottom	9.0	+160	22	39	
	3 –5	138	...	350	42	Top	34.0	+480	83	10.3	 1 Web — 40 x 1 2 C. Pl.— 40 x 0.8 2 ∟s — 9 x 9 x 1
	3′–5′	138	...	350	42	Bottom	7.6	+350	11	46	
	5 –6	138	...	350	42	Top	34.0	+480	87	10.3	
	5′–6′	138	...	350	42	Bottom	7.6	+350	11	46	
	6 –8	178	...	350	43	Top	35.8	+450	111	9.8	 1 Web — 40 x 1 2 C. Pl.— 40 x 0.8 1 C. Pl.— 40 x 1 2 ∟s — 9 x 9 x 1
	6′–8′	178	...	350	43	Bottom	6.8	+400	10	52	
	8 –9	178	...	350	43	Top	35.8	+450	45	9.8	
	8′–9′	178	...	350	43	Top	35.8	+400	13	9.8	
Top Chord	2 –4	106	14600	700	41	Bottom	31.8	−480	21	22	 1 Web — 40 x 1 1 C. Pl.— 40 x 0.8 2 ∟s — 9 x 9 x 1
	2′–4′	106	14600	700	41	Top	9.0	−300	13	78	
	4 –7	162	18100	700	42.2	Top	6.9	−480	8	101	 1 Web — 40 x 1 1 C. Pl.— 40 x 1.4 1 C. Pl.— 40 x 0.8 2 ∟s — 9 x 9 x 1
	4′–7′	162	18100	700	42.2	Top	6.9	−350	9	101	
	7 –7′	182	18450	700	43.2 $b = 2e$	Top	7.1 $e_1 = e_2$	−400	10	99 $\frac{l}{b}$	 1 Web — 40 x 1 1 C. Pl.— 40 x 1.4 1 C. Pl.— 40 x 0.8 1 C. Pl.— 10 x 1 2 ∟s — 9 x 9 x 1
Diagonals	2 –3	93.6	...	712	35	...	17.5	+460	11	20	 1 Web — 85 x 1 2 C. Pl.— 17.5 x 1 2 ∟s — 7 x 7 x 0.9
	2′–3′	93.6	...	712	35	...	17.5	+320	17	20	
	4 –6	58.8	...	712	26	...	13.0	+340	18	27	 1 Web — 26 x 1 2 ∟s — 8 x 8 x 1.1
	4′–6′	58.8	...	712	26	...	13.0	+400	15	27	
	7 –9	38.8	...	712	17	...	8.5	−260	27	42	 2 ∟s — 8 x 8 x 1.1
	7′–9	38.8	...	712	17	...	8.5	+530	11	42	
	0 –2	114.8	...	712	40	...	20.0	−520	44	18	 1 Web — 40 x 1 2 C. Pl.— 20 x 1 2 ∟s — 8 x 8 x 1.1
	0′–2′	114.8	...	712	40	...	20.0	−300	30	18	
	3 –4	87.6	...	712	32	...	16.0	−380	26	22	 1 Web — 32 x 1 2 ∟s — 7 x 7 x 0.9
	3′–4′	87.6	...	712	32	...	16.0	−310	39	22	
	6 –7	52.1	...	712	21	...	10.5	−150	47	34	 2 ∟s — 10 x 10 x 1.4
	6′–7′	52.1	...	712	21	...	10.5	−420	14	34	
Verticals	1 –2	21.1	...	600	15	...	8.0	+590	12	40	 2 ∟s — 7 x 7 x 0.8
	4 –5	21.1	...	600	15	...	8.0	+420	17	40	
	7 –8	21.1	...	600	15	...	8.0	+575	5	40	
	7′–8′	21.1	...	600	12	...	8.0	+100	90	40	
	4′–5′	21.1	...	600	15	...	8.0	+100	60	40	
	1′–2′	21.1	...	600	15	...	8.0	+100	65	40	

Secondary stresses are calculated for gross moments of inertia.

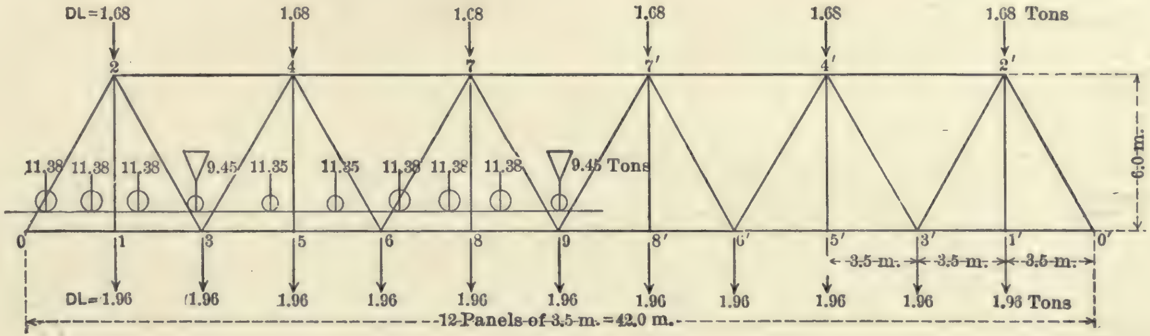


Fig. 48.

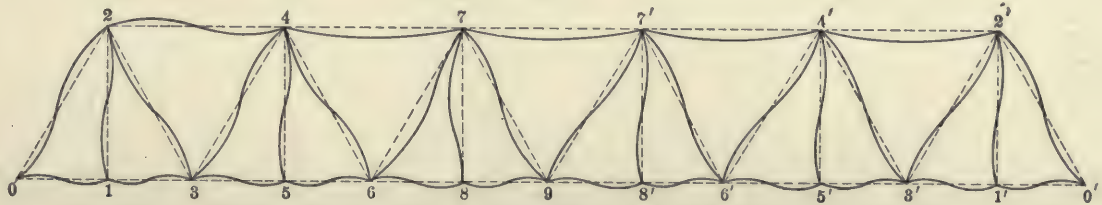


Fig. 48 A.

Through Warren Truss with Suspenders of a Railroad Bridge across the river Birs near Mönchenstein in Switzerland. Bridge collapsed in June, 1891.

Calculated by Professor Ritter and taken from "Schweizerische Bauzeitung," 1891.

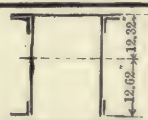
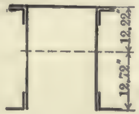
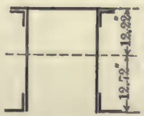
Secondary stresses determined for dead load and positions of live load as indicated.

										$\frac{I}{b}$	
Bottom Chord	0–2	47.36	3888	324	24.50	12.25	...	+11500	32	13	2 Webs — 24" x ½" 4 Ls — 4" x 4" x 13/16"
	2–4	47.36	3888	324	24.50	12.25	...	+11500	29	13	2 Webs — 24" x ½" 4 Ls — 4" x 4" x 13/16"
	4–6	83.36	5508	324	24.50	12.25	...	+12000	43	13	2 Webs — 24" x 1⅛" 4 Ls — 4" x 4" x 13/16"
Diagonals	1–4	47.36	3888	540	24.50	12.25	...	+11500	10	22	2 Webs — 24" x ½" 4 Ls — 4" x 4" x 13/16"
	4–5	30.24	1296	540	18.50	9.25	...	−5000	22	22	2 Webs — 18" x ½" 4 Ls — 3" x 3" x 7/16"
Verticals	1–2	22.48	504	432	16.75	8.38	...	+11200	19	26	2 Bulb Ls 8" x 3½" x 7/16"
	3–4	22.48	504	432	16.75	8.38	...	−1200	201	26	
	5–6	22.48	504	432	16.75	8.38	...	+11200	...	26	

Secondary stresses are calculated for gross moments of inertia.

TABLE 6 AND FIG. 49.

Span 162 Feet. Double Track. Steel.

Member.		Gross area, sq. in.	I gross	l In.	b In.	e In.	Most dangerous fiber.	Stresses. lbs./sq. in.	Stresses. %	$\frac{l}{e}$	Sections.
End Post	0–1	84.02	6750	540	24.94	12.32	Top	−8900	18	42	 1 C. Pl.—24" x 7/16" 2 Webs—24" x 1⅛" 2 L^s—3" x 3" x ½" 2 L^s—6" x 4" x ¾"
Top Chord	1–3	69.02	6000	324	24.94	12.22	Top	−10400	20	27	 1 C. Pl.—24" x 7/16" 2 Webs—24" x 13/16" 2 L^s—3" x 3" x ½" 2 L^s—6" x 4" x ¾"
	3–5	69.02	6000	324	24.94	12.22	Top	−10400	17	27	 1 C. Pl.—24" x 7/16" 2 Webs—24" x 13/16" 2 L^s—3" x 3" x ½" 2 L^s—6" x 4" x ¾"

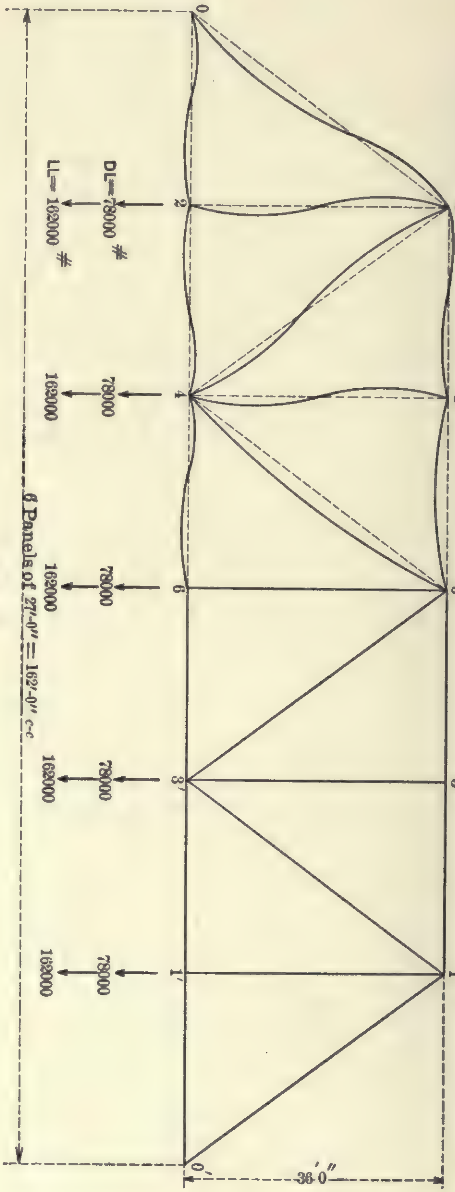


Fig. 49.

Through Warren Truss with Suspenders of a Double Track American Railroad Bridge. Calculated by the Author.
Secondary stresses determined for dead load and positions of live load as indicated. —

by the dimensions of each bar and the distribution of its material. It is for this reason that each table contains a column, which gives the ratio between length and width of each bar having a symmetrical section, and for bars with unsymmetrical sections the ratio between length and distance of outer fiber from the neutral axis is given, and with a few exceptions the tables show also the distribution of material in the sketches of the sectional areas.

We will now point out some facts which indicate how different trusses are affected by secondary stresses. An inspection of the Tables II, III and IV, with Figs. 45, 46 and 47, shows that the secondary stresses of the chords in single Warren trusses without verticals increase from the center of the span toward the end, while for the web members these stresses increase in the opposite direction. This circumstance is in so far fortunate as the web members in the neighborhood of the center of the span have often a surplus in section and the end members of both the top and bottom chords show this surplus still oftener.

The secondary stresses in Warren trusses with verticals, Fig. 48, Table V, and Fig. 49, Table VI, do not entirely bear out the statements made in reference to the Warren trusses without verticals. It is the bottom chord which shows in both cases irregularities not observed in Warren trusses without verticals. The run of the secondary stresses in Fig. 48 and Table V, can easily be accounted for by the fact that the live load covers only one-half of the span.

If verticals carry a heavy concentrated load, then they elongate a good deal. These elongations are the cause of considerable deformations in the bottom chord members accompanied by severe secondary stresses.

The bottom chord member 4–6 in Fig. 49, shows the greatest secondary stress to be 43 per cent of the primary stress, while in the members 1–3 of Fig. 48, it is as high as 143 per cent. In comparing these secondary stresses with respect to the unit primary and total stresses, we find that the secondary stress in the member 1–3 of Fig. 48 amounts to 5690 pounds per square inch, which

with the primary stress of 3980 pounds per square inch, results in 9670 pounds per square inch, while in the member 4–6 of Fig. 49, the secondary stress is 5160 pounds per square inch, making a total of 12,000 + 5160 = 17,160 pounds per square inch. But the bottom chord section 6–8 of Fig. 48, with an increase of stress of 111 per cent or 7100 pounds per square inch shows a primary stress of 6400 pounds per square inch, which gives a total of 13,500 pounds per square inch or a stress considerably higher than in the member 1–3 of the same truss.

Of course the idea presents itself as natural that the reduction of these high secondary stresses is easily affected by providing the verticals with ample sectional areas, because in so doing we reduce their elongations and consequently the deformations of the bottom chord.

Excluding the truss Fig. 51 and Table VIII from a comparison on account of the incompleteness of the data, we find the run of the secondary stresses in the two Pratt trusses Fig. 50, Table VII, and Fig. 52, Table IX, in a rather close agreement, although these stresses have been determined under varying conditions and such differences as do exist can be explained.

The secondary stresses for the truss Fig. 50 and Table VII, have been calculated for one single position of the live load, while those for the truss Fig. 52 and Table IX, have been found by the method of influence lines. Sometimes the writer reversed the direction of the train, but he always placed the second driver of the first engine at any one of the panel points, so that the stresses given are not the absolute greatest.

The secondary stresses from the center of the span along the top chord and the end post in Fig. 50 first increase and then decrease, and this is also true for the truss, Fig. 52, up to the intersection point between end post and collision strut, the maximum stress being reached in the lower fragment of the end post. This maximum is due to the collision strut, which by its division of the end post into two fragments decreases the ratio $\frac{l}{\varepsilon}$ and has, there-

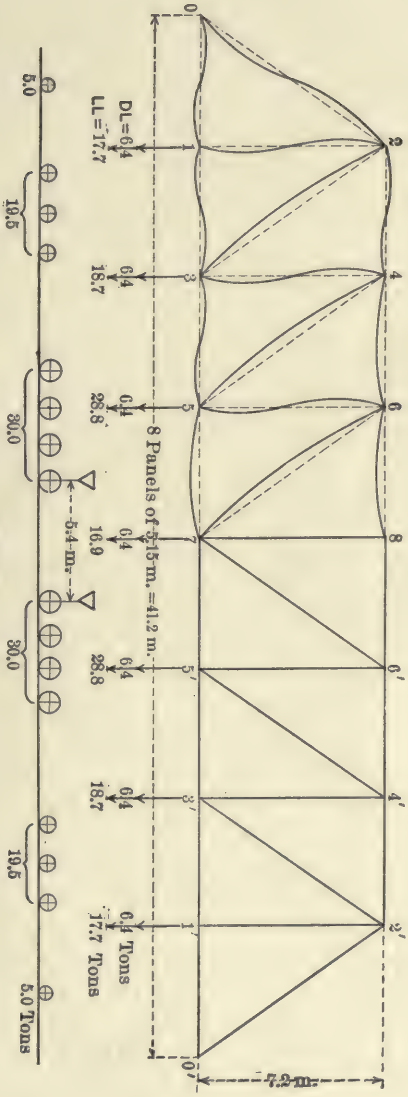
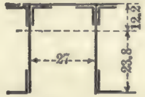
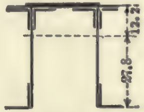
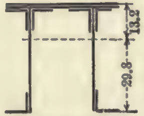
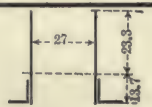
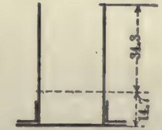
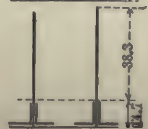


Fig. 50.

Through Pratt Truss of a Russian Railroad Bridge. Calculated by Professor Patton. Secondary stresses determined for dead load and positions of live load as indicated.

Diagonals	0–2	207	35075	886	37	...	$e_1 = e_2$	−527	4	4	$\frac{1}{b}$	27, 21.3, 12.7	2 Webs—36 x 1 1 C. Pl.—45 x 1 6 L^s—8 x 8 x 1
	2–3	119	4018	886	27	...	13.5	+632	3	3	33	27	4 L^s 13 x 13 x 13
	4–5	83	3217	886	27	...	13.5	+620	5	5	33	27	4 L^s 13 x 8.5 x 1
	6–7	83	3217	886	27	...	13.5	+141	28	27	33	27	4 L^s—13 x 8.5 x 1
Verticals	1–2	56	686	720	16	...	8	+376	23	22	45	16	4 L^s—7.5 x 7.5 x 1
	3–4	148	5825	720	27	...	13.5	−286	63	56	27	27	4 L^s—13 x 6.5 x 1
	5–6	113	1371	720	16	...	8	−88	75	72	45	16	4 L^s—7.5 x 7.5 x 1
	7–8	56	686	720	16	...	8	0	...	...	..	16	4 L^s—7.5 x 7.5 x 1

TABLE 7 AND FIG. 50. Span 41.2 m. Steel.

Member.		Gross area, sq.cm.	I net cm^4.	l cm.	b cm.	Most dangerous fiber	e cm.	Stresses. Kg. sq. cm.	Stresses. % I net.	Stresses. % I gross.	$\frac{l}{e}$	Sections. Dimensions in cm.	
Top Chord	2–4	205	32912	515	36	Top	12.2	−546	7	7	42		2 Webs—35 x 1 1 C. Pl.—45 x 1 6 L^s—8 x 8 x 1
	4–6	242	47743	515	41	Top	13.2	−590	15	14	39		2 Webs—39 x 1 1 C. Pl.—46 x 1 1 C. Pl.—29 x 1 6 L^s—8 x 8 x 1
	6–8	262	56018	515	43	Top	13.2	−575	12	11	39		2 Webs—41 x 1 2 C. Pl.—45 x 1 6 L^s—8 x 8 x 1
Bottom Chord	0–1	104	13487	515	37	Bottom	13.7	+610	18	16	38		2 Webs—37 x 1 2 L^s—8 x 8 x 1
	1–3	104	13487	515	37	Bottom	13.7	+610	16	15	38		
	3–5	171	40143	515	49	Bottom	14.7	+640	17	16	35		2 Webs—48 x 1 1 C. Pl.—45 x 1 2 L^s—8 x 8 x 1
	5–7	209	53166	515	53	Bottom	14.7	+670	12	11	35		2 Webs—52 x 1 1 C. Pl.—45 x 1 4 L^s—8 x 8 x 1

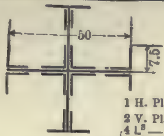
Verticals	0–1	2	615	50	25	−182	97	12		1 H. Pl. 50 x 1 2 V. Pl. 25 x 1 4 L^s—10 x 10 x 1 8 L^s—7.5 x 7.5 x 1
	2–3	2	615	40	20	−277	202	15		1 H. Pl. 40 x 1 2 V. Pl. 15 x 1 4 L^s—12 x 8 x 1.1
	4–5	4	615	35	17.5	−252	124	18		1 H. Pl. 35 x 1 2 V. Pl. 13.5 x 1 4 L^s—12 x 8 x 1
	6–7	8	615	30	15	−217	82	21		1 H. Pl. 30 x 1 2 V. Pl. 11 x 1 4 L^s—7 x 7 x 1.1
	8–9	10	615	25	12.5	−182	79	25		1 H. Pl. 25 x 1 2 V. Pl. 11 x 1 4 L^s—9.5 x 6 x 1.1

TABLE 8 AND FIG. 51.

Span 54.5 m. Wrought Iron.

Member.		Position of first wheel at panel point.	l cm.	b = 2e cm.	$e_1 = e_2$ cm.	Stresses. Kg./sq. cm.	Stresses. %	$\frac{l}{b}$	Sections in cm.
Diagonals	1–2	2	818	40	20	+464	35	21	 1 H. Pl. 40 x 1 2 V. Pl. 14.5 x 1 4 L — 17.5 x 9 x 1
	3–4	4	818	35	17.5	+445	42	23	 1 H. Pl. 35 x 1 2 V. Pl. 13.5 x 1 4 L — 12 x 8 x 1
	5–6	6	818	30	15	+431	46	27	 1 H. Pl. 30 x 1 2 V. Pl. 11 x 1 4 L — 7.5 x 7.5 x 1
	7–8	8	818	25	12.5	+339	26	33	 1 H. Pl. 25 x 1 2 V. Pl. 11 x 1 4 L — 7 x 7 x 1
	9–10	10	818	27	13.5	+245	36	30	 1 H. Pl. 27 x 1 2 V. Pl. 14 x 1 4 L — 7 x 7 x 1

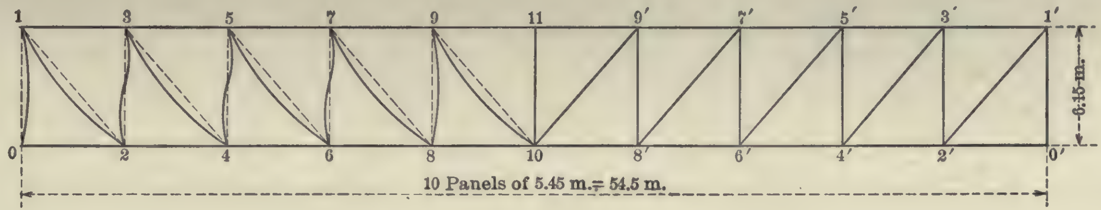
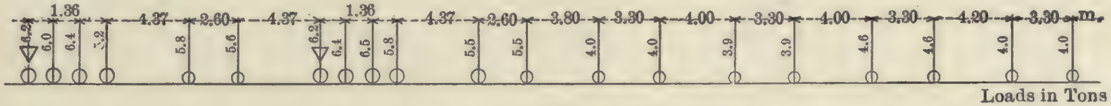


Fig. 51.



Pratt Truss of a Railroad Bridge across the river Iste, near Cognac, on the Orleans Railroad in France.

The secondary stresses have been measured by means of instruments (system Rabut) and are taken from experiments published by engineer Ménagé in "Annales des ponts et chaussées," 1899. — They are given for verticals and diagonals only and are exclusively due to the live load, consisting of two engines and cars. The positions of the live load are given in Table 8. The secondary stresses correspond to the maximum primary stresses.

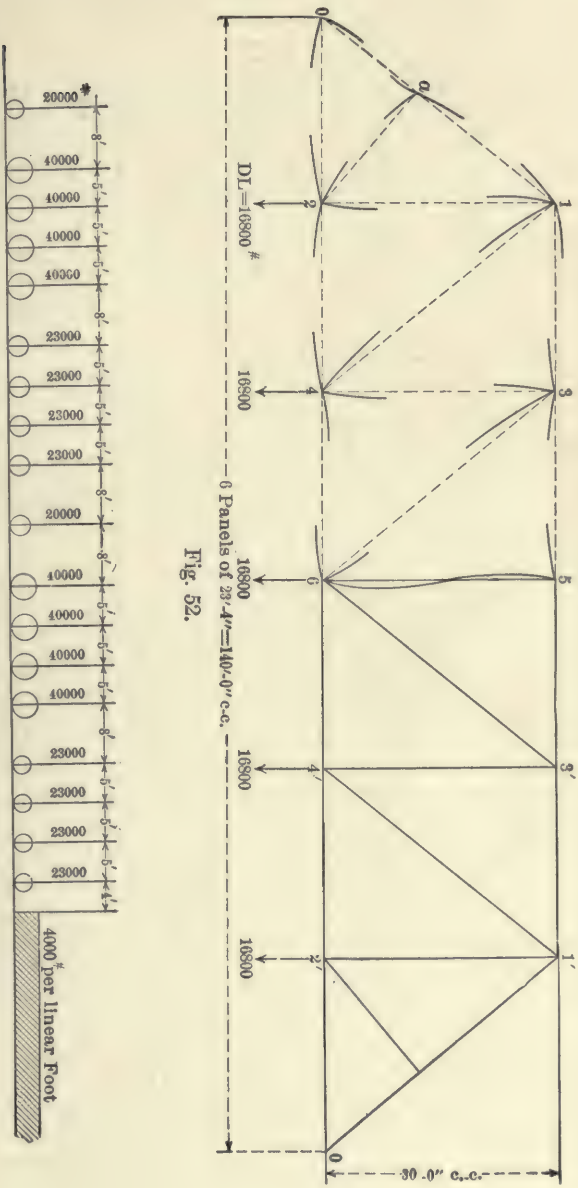


Fig. 52.

Through Pratt Truss of a Single Track American Railroad Bridge. At present in service.
Secondary stresses calculated by the author by means of influence lines.
Dead load and live load as indicated.—Deformations shown are due to the combined influence of $DL + LL$ for different positions of the train.

Diagonals	1–4	20.58	360	456	12	6	...	3700	11	38	2 – 12″ ⊔ – 35 #	2 Right
	3–6	14.70	288	456	12	6	...	4330	19	38	2 – 12″ ⊔ – 25 #	4 Left
Verticals	1–2	13.68	120	360	12.75	6.375	...	2500	34	28	4 L^s – 6″ x 3½″ x 3/8″	4 Left
	3–4	14.70	288	360	12	6	...	5160	30	30	2 – 12″ ⊔ – 25 #	6 Left
	5–6	14.70	288	360	12	6	...	...	610# / sq.in.	30	2 – 12″ ⊔ – 25 #	4′ Left
Collision Strut	2–a	9.92	36.9	221	8.75	4.375	...	...	530# / sq.in.	25	4 L^s – 4″ x 3″ x 3/8″	2 Left

Secondary stresses are calculated for gross moments of inertia.

TABLE 9 AND FIG. 52.

Span 140 Feet. Single Track. Medium Steel.

Member.		Gross area, sq. in.	I gross.	l in.	b in.	e in.	Most dangerous fiber.	Stresses. lbs./sq. in.	Stresses. %	$\frac{l}{e}$	Sections.	Train Heading to Left or Right. Position of 2d Driver at Panel-point.
End Post	0–a	35.66	1154	172	15.44	5.94	Top	7370	31	29	 2—15" ⊔—45#, 1 C. Pl.—21" x 7/16"	2 Left
	1–a	35.66	1154	284	15.44	9.5	Bottom	7370	23	30		2 Left
Top Chord	1–3	28.98	1000	280	15.44	9.9	Bottom	8640	29	28	 2—15" ⊔—33#, 1 C. Pl.—21" x 7/16"	2 Left
	3–5	32.70	1080	280	15.44	5.74	Top	8650	10	49	 2—15" ⊔—40#, 1 C. Pl.—21" x 7/16"	2 Left
Bottom Chord	0–2	18.00	174	280	12.75	6.375	...	11500	9	22	 4 L^s—6" x 3½" x ½"	2 Left
	2–4	18.00	174	280	12.75	6.375	...	9300	11	22		4 Left
	4–6	29.37	335	280	13	6.5	...	7490	26	22	 4 L^s—6" x 3½" x ½", 2 Pl.—13" x 7/16"	6 Left

TABLE 10 AND FIG. 53.

Span 27 m. Wrought Iron.

Member.		Gross area, sq. cm.	I gross cm^4.	l cm.	b cm.	Most danger-ous fiber.	e cm.	Stresses. Case I. Kg./sq. cm.	Case I. %	Case II. Max. Kg./sq. cm.	Case II. %	$\frac{l}{e}$	Sections.
Top Chord	1–3	60	2800	450	23	Bottom	15.8	0	...	−12.5	50	28	
	3–5	80	3700	450	24	Bottom	18.7	−18.8	83	−28.1	56	24	
	5–7	100	4500	450	25	Bottom	21.1	−15.0	177	−30.0	88	21	
Bottom Chord	0–2	60	2800	450	23	Top	15.8	+18.8	41	+18.8	41	28	Sections consist of 1 web, 2 angles and 1 or 2 or no cover plates
	2–4	80	3700	450	24	Top	18.7	+14.1	164	+32.8	70	24	
	4–6	100	4500	450	25	Top	21.1	+18.8	127	+33.8	71	21	
Diagonals	1–2	36	2100	750	b = 2e 26	...	$e_1 = e_2$ 13	0	...	+34.7	12	$\frac{l}{b}$ 29	Symmetrical
	3–4	28	900	750	20	...	10	+22.3	23	+22.3	23	37	
	5–6	43	400	750	14	...	7	0	...	+ 9.7	17	54	
	0–3	106	2500	750	24	...	12	−17.7	12	−18.3	12	31	
	2–5	96	1700	750	20	...	10	0	...	−13.0	22	37	
	4–7	80	1100	750	18	...	9	−7.8	33	−10.4	25	42	
End Post	0–1	96	1400	600	16	...	8	0	...	−10.4	24	37	

Secondary stresses are calculated for gross moments of inertia.

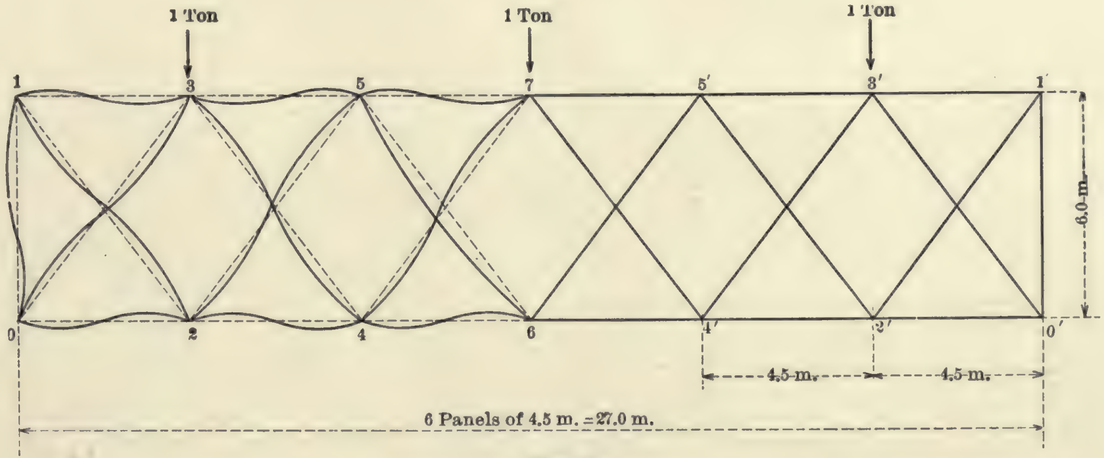


Fig. 53.

Double Intersection Warren Bridge Truss. Calculated by Professor Winkler and taken from his book "Theorie der Brücken," II Heft.

The diagonals are not riveted at their intersection points.

The calculations have been made for two cases of loading. In case I the primary and secondary stresses were calculated for the positions of the loads as indicated. These positions correspond to the maximum secondary stresses in the chords. In case II the positions of the loads were changed so as to give the maximum primary stresses, while the secondary stresses were determined for the positions of the loads as indicated.

TABLE 11 AND FIG 54.

Span 27 m. Wrought Iron.

Member.		Gross area, sq. cm.	I gross cm^4.	l cm	b cm	Most danger-ous fiber.	e cm.	Stresses. Case I. Kg. / sq. cm.	Case I. %	Case II. Max. Kg. / sq. cm.	Case II. %	$\frac{l}{e}$	Sections.
Top Chord	1–3	60	2800	450	23	Top	7.2	0	...	−12.5	27	62	
	3–5	80	3700	450	24	Bottom	18.7	−18.8	55	−28.1	36	24	
	5–7	100	4500	450	25	Bottom	21.1	−15.0	227	−30.0	113	21	
Bottom Chord	0–2	60	2800	450	23	Top	15.8	+18.8	73	+18.8	73	29	Sections consist of 1 web, 2 angles and 1 or 2 or no cover plates
	2–4	80	3700	450	24	Top	18.7	+14.1	102	+32.8	44	24	
	4–6	100	4500	450	25	Top	21.1	+18.8	115	+33.8	64	21	
Diagonals					$b = 2e$		$e_1 = e_2$					$\frac{l}{b}$	Symmetrical
	1–2	36	2100	375	26	...	13	0	...	+34.7	28	14	
	3–4	28	900	375	20	...	10	+22.3	56	+22.3	56	19	
	5–6	43	400	375	14	...	7	0	...	+ 9.7	90	27	
	0–3	106	2500	375	24	...	12	−17.7	29	−18.3	28	16	
	2–5	96	1700	375	20	...	10	0	...	−13.0	51	19	
	4–7	80	1100	375	18	...	9	−7.8	100	−10.4	75	21	
End Post	0–1	96	1400	600	16	...	8	0	...	−10.4	23	37	

Secondary stresses are calculated for gross moments of inertia.

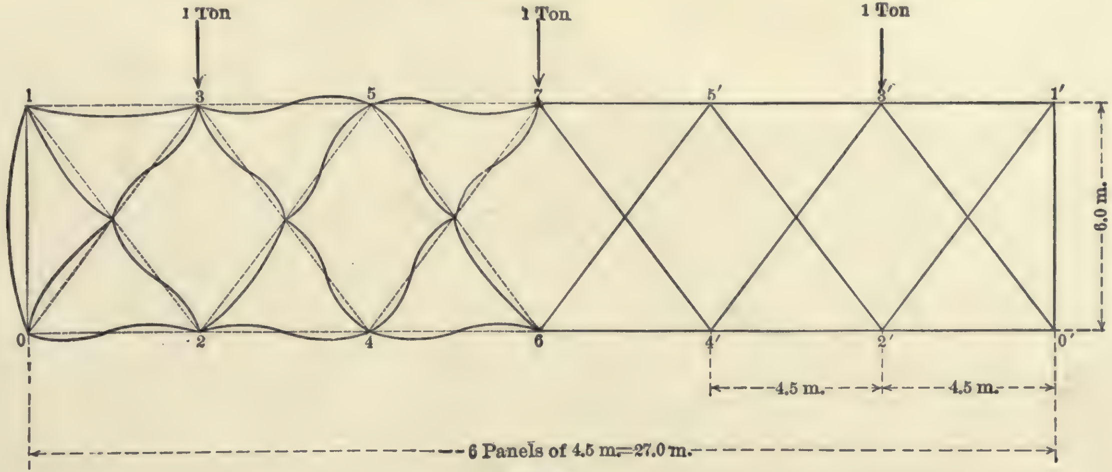


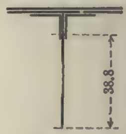
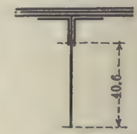
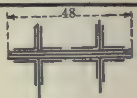
Fig. 54.

Double Intersection Warren Bridge Truss. Same as Fig. 53. Calculated by Professor Winkler and taken from his book "Theorie der Brücken," II Heft.

The conditions for the loading in this case are the same as those for Fig. 53, excepting that the diagonals are riveted at their intersection points.

TABLE 12 AND FIG. 55.

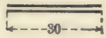
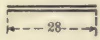
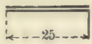
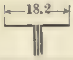
Span 40 m. Wrought Iron.

Member.		l cm.	b cm.	Most danger-ous fiber.	e cm.	Case I. Stresses, when diagonals are riveted. Kg./sq. cm.	%	Case II. Stresses, when diagonals are not riveted. Kg./sq. cm.	%	l/e	Sections in cm.
Top Chord	1–3	400	46.2	Bottom	36.5	−150	117	−150	120	11	 1 Web — 45 x 1.5 1 C. Pl.— 48 x 1.2 2 L^s—11.8 x 11.8 x 1.3
	3–5	400	46.2	Top	9.7	−390	12	−390	12	41	
	5–7	400	47.4	Top	8.6	−460	12	−460	12	46	 1 Web — 45 x 1.5 2 C. Pl. — 48 x 1.2 2 L^s—11.8 x 11.8 x 1.3
	7–9	400	48.6	Top	8.0	−500	7	−500	7	50	 1 Web — 45 x 1.5 3 C. Pl. — 48 x 1.2 2 L^s—11.8 x 11.8 x 1.3
	9–11	400	48.6	Top	8.0	−540	8	−540	8	50	
Bottom Chord	0–2	400	46.2	Top	36.5	+210	145	+210	117	11	same as for 1–3 and 4–5
	2–4	400	46.2	Bottom	9.7	+480	7	+480	8	41	
	4–6	400	47.4	Bottom	8.6	+530	8	+530	10	46	same as for 5–7
	6–8	400	48.6	Bottom	8.0	+560	7	+560	6	50	same as for 7–9 and 9–11
	8–10	400	48.6	Bottom	8.0	+590	7	+590	6	50	
Verticals										l/b	
	0–1	400	48	...	...	−105	133	−105	124	8	 1 Pl.—48 x 1.5 2 Pl.— 48 x 1.3 2 Pl.— 11 x 1 4 L^s—10.5 x 10.5 x 1.2 4 L^s—9.2 x 9.2 x 1.2
	2–3	400	17	...	...	−30	483	−30	513	24	
	4–5	400	17	...	...	−50	220	−50	220	24	
	6–7	400	17	...	...	−60	141	−60	141	24	
	8–9	400	17	...	...	−70	50	−70	71	24	
	10–11	400	17	...	...	−90	0	−90	0	24	4 L^s—8 x 8 x 1

Secondary stresses are calculated for gross moments of inertia.

TABLE 13 AND FIG. 55.

Span 40 m. Wrought Iron.

	Member.	Gross area, sq. cm.	l cm.	b cm.	$e_1 = e_2$ cm.	Case I. Stresses, when diagonals are riveted. Kg./sq. cm.	%	l/b	Case II. Stresses, when diagonals are not riveted. Kg./sq. cm.	%	l/b	Sections in cm.
Diagonals descending to the right	1–2	84	566	30	15	+543	23	9.4	+543	11	19	 2 Fl. 30 x 1.4
	3–4	73	566	28	14	+500	22	10	+500	20	20	 2 Fl. 28 x 1.3
	5–6	60	566	25	12.5	+420	23	11	+420	17	23	 2 Fl. 25 x 1.2
	7–8	60	566	25	12.5	+235	34	11	+235	25	23	
	9–10	49	566	18.2	9.1	+ 90	78	16	+ 90	33	31	 1 Fl. 9.5 x 1.2 2 L^s – 8.5 x 8.5 x 1.2
Diagonals descending to the left	0–3	95	566	21	10.5	−580	16	13	−580	9	27	 4 L^s – 10.5 x 10.5 x 1.2
	2–5	85	566	19	9.5	−500	14	15	−500	14	30	 4 L^s – 9.5 x 9.5 x 1.2
	4–7	72	566	19	9.5	−440	16	15	−440	11	30	 4 L^s – 9.5 x 9.5 x 1
	6–9	72	566	19	9.5	−275	24	15	−275	18	30	
	8–11	49	566	18.2	9.1	−155	40	16	−155	16	31	 1 Fl. 9.5 x 1.2 2 L^s – 8.5 x 8.5 x 1.2

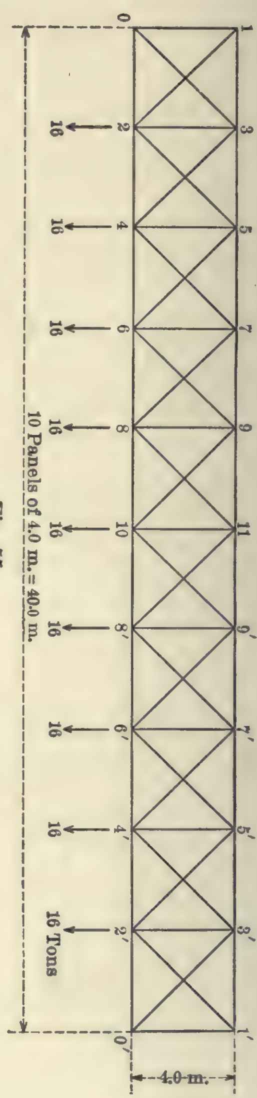


Fig. 55.

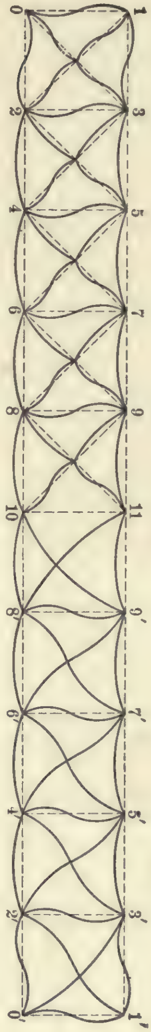


Fig. 55 A.

Double Intersection Through Warren Truss with Verticals of the Inschialpbach Bridge of the Gotthard Railroad. Calculated by Professor Ritter and taken from his book "Anwendungen der graphischen Statik," II Theil. Secondary stresses determined for positions of loads as indicated.

fore, no beneficial influence on the end post as far as secondary stresses are concerned.

The run of the secondary stresses in the web members for both trusses is very nearly the same, increasing from the end toward the center of the span.

The vertical 5–6 in Fig. 52 has been assumed as carrying no dead weight, otherwise the percentage of secondary stress would be very high.

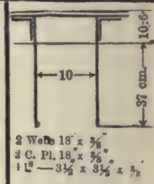
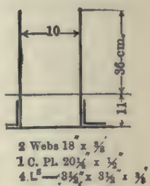
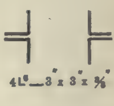
Comparing the secondary stresses in the bottom chords of Fig. 50 and Fig. 52, we see that they decrease in Fig. 50 from the end toward the center of the span, while they increase in Fig. 52 in the same direction. This phenomenon is attributable to the collision strut, which is wanting in Fig. 50. This strut resists the hip vertical in its elongation and consequently lessens the deformations of the bottom chord sections 0–2 and 0–4.

The middle vertical 5–6 in Fig. 52, and the collision strut experience both a reversal of stresses and this is quite natural, as these members are supposed to carry neither dead nor live load stresses.

The double intersection Warren truss without verticals, Fig. 53 and Table X, as also Fig. 54 and Table XI, shows rather severe secondary stresses, which increase from the end of the span towards the center and this direction is contrary to that shown by the single Warren truss. The reason for these high secondary stresses is that for unequal loading of the single systems which compose the truss, the chords are subjected to rather large deformations. These latter, and consequently the secondary stresses, can be reduced by the insertion of verticals, which effectively connect the two single systems. The truss shown in Fig. 55 and Tables XII and XIII proves this clearly. The secondary stresses in the chords of these trusses increase from the center of the span toward its end and those of the diagonals increase from the end toward the center of the span, which is the same run as found in single Warren trusses. The stresses in the diagonals also increase when they are riveted at their intersection points.

TABLE 14 AND FIG. 56.

Span 27.02 m. Steel.

	Member.	Gross area, sq. cm.	I gross cm.[4]	l cm.	b cm.	Most dangerous fiber.	e cm.	$\frac{l}{e}$ cm.	Case I. Stresses. For each uniform total loading. % I net.	Case I. Stresses. For each uniform total loading. % I gross.	Case II. Stresses. For the most unfavorable positions of the loads. Max. Kg. sq. cm.	Case II. Stresses. For the most unfavorable positions of the loads. % I net.	Sections.
Top Chord	0–1	239	43736	386	47.6	Top	10.6	37	33	28	−591	27	
	1–3	239	43736	386	47.6	Top	10.6	37	29	25	−582	31	
	3–5	239	43736	386	47.6	Top	10.6	37	7	6	−559	7	
	5–5′	239	43736	386	47.6	Top	10.6	37	7	6	−557	5	
Bottom Chord	0–2	216	40757	213	47	Bottom	11	19	40	33	+658	42	
	2–4	216	40757	410	47	Bottom	11	37	36	30	+714	38	
	4–6	216	40757	395	47	Bottom	11	45	8	7	+651	6	
	6–8	216	40757	387	47	Bottom	11	35	8	7	+632	7	
Diagonals							$e_1 = e_2$	$\frac{l}{b}$					
	1–2	55	688	213	16	Bottom	8	13	60	47	−332	54	
	1–4	55	688	298	16	Top	8	19	76	59	−432	86	
	3–4	55	688	298	16	Bottom	8	19	80	62	−340	52	
	3–6	55	688	365	16	Top	8	23	28	22	−459	23	
	5–6	55	688	365	16	Bottom	8	23	35	27	−396	18	
	5–8	55	688	389	16	Top	8	24	49	38	−436	27	

The direct stresses and the sums of the direct and secondary stresses have been each calculated for the most unfavorable positions of the live load.

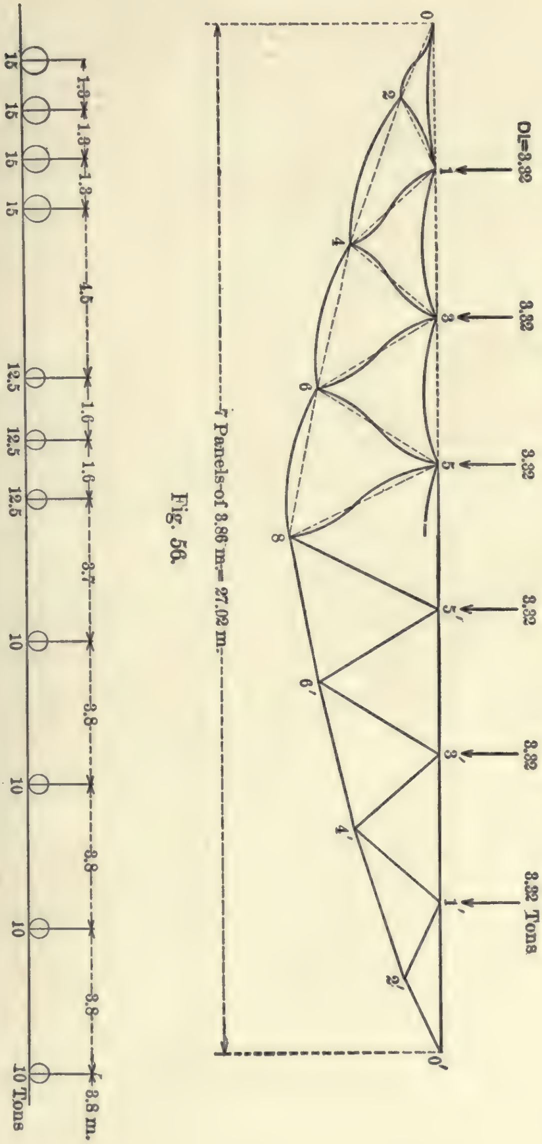
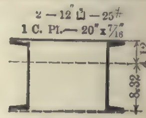
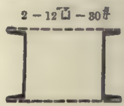
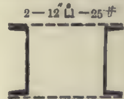
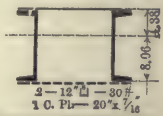
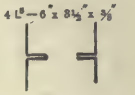
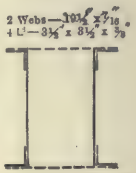


Fig. 56.

Parabola Truss of a Russian Railroad Bridge. Calculated by Professor Patton by means of influence lines.
Dead load and live load as indicated.

TABLE 15 AND FIG. 57.

Draw Span 202 Feet. Single Track. Steel.

Member.		Gross area, sq. in.	I gross.	l in.	b in.	e in.	Most danger-ous fiber.	Stresses. lbs. / sq. in.	Stresses. %	l/e	Sections.
End Post	0–1	23.45	500	452	12.44	4.12	Top	−6080	8	109	
Top Chord	1–3	23.45	500	303	12.44	8.32	Bottom	−4300	22	36	
	3–5	23.45	500	303	12.44	4.12	Top	−4300	78	73	
	5–7	20.58	359	318	12	6		+7210	80	l/b 26	
Bottom Chord	0–2	17.64	323	303	12	6	...	+6120	4	25	
	2–4	17.64	323	303	12	6	...	+6120	16	25	
	4–6	17.64	323	303	12	6	...	+1020	1186	25	
	6–8	17.64	323	303	12	6	...	+1020	868	25	
Diagonals	1–4	14.70	288	452	12	6	...	+ 610	190	38	
	4–5	17.64	323	452	12	6	...	+8120	60	38	
	5–8	26.40	550	452	12.44	4.38	Top	−7900	40	l/e 103	
Verticals	1–2	13.68	120	336	12.38	6.19	...	+8200	3	l/b 27	
	3–4	13.68	120	336	12.38	6.19	...	...	1280# sq. in.	27	
	5–6	13.68	120	336	12.38	6.19	...	+8200	110	27	
	7–8	28.54	1475	432	20	10	...	−2750	...	...	

Secondary stresses are calculated for gross moments of inertia.

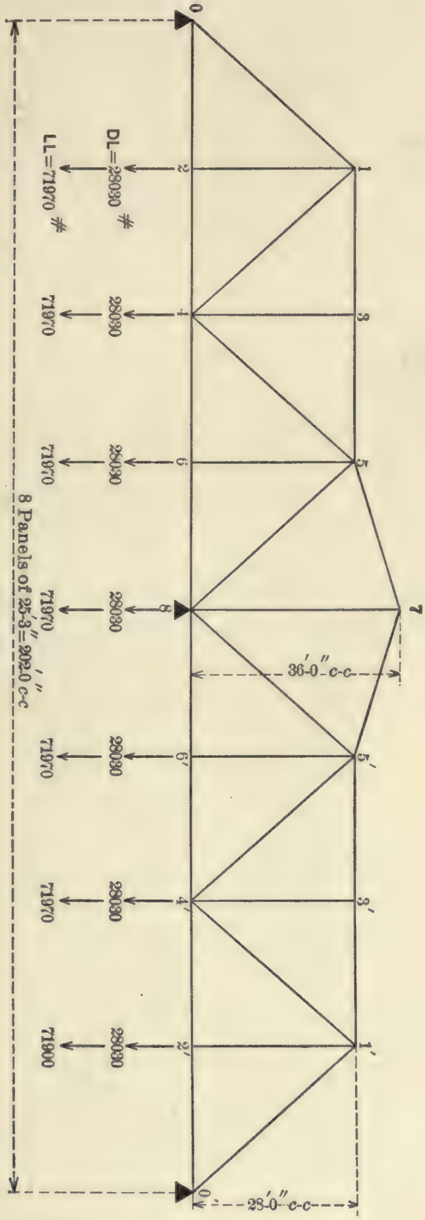


Fig. 57.

Truss of a Single Track American Draw Bridge Calculated by the Author
Secondary stresses determined for dead load and positions of live load as indicated.

The parabola truss, Fig. 56 and Table XIV is a Russian railroad bridge with the wooden ties resting directly on the top chords. The ratios between total stress and primary stress remain constant for any uniformly distributed load.

The single Warren truss with verticals, Fig. 57 and Table XV, is continuous over three supports and intended for a drawbridge. The writer determined the secondary stresses for "bridge closed," the truss resting on three supports and covered with the live load from end to end. The center reaction has been taken as the unknown quantity and calculated by means of the principle of the derivative of work. After this reaction was found the primary stresses were calculated.

Owing to the fact that this truss is continuous over three supports, the run of the secondary stresses is contrary to that in single Warren trusses on two supports. We see that they increase in the chords from the end of the span toward the center and decrease in the diagonals in the same direction. High secondary stresses can be expected at the center of the span and in its neighborhood, no matter whether the bridge is closed or open, as it is at these places where the deformations are the greatest, and they are the smallest in the end panel with inconsiderable bending stresses when the bridge is open.

An increase in the stress of the bottom chord section 4–6 of 1186 per cent is very severe, but we must not forget that its primary stress is very small, amounting only to 1020 pounds per square inch. The primary stress in the end bottom chord sections is six times greater than that in the bars 4–6 and 6–8 and consequently offers a greater resistance to the deformations at the panel point 2. The total stresses per square inch in the bottom chord sections from the end of the span to the center are 6360, 9790, 13,120 and 9870 pounds, while the primary stresses amount to 6120 and 1020 pounds per square inch.

We will now enumerate some points which are guiding for a designer to minimize secondary stresses due to static loads. The effects of impact, vibrations, derailments and collisions are beyond

an analysis, but they should not be overlooked because they too are of a secondary nature.

It has already been mentioned that secondary stresses decrease if the ratio between the length of a bar and its width increases. This means that members of great circumference and shortness are more susceptible to secondary stresses than long and slender members, a fact already noted when we spoke of secondary stresses in cross frames. In this respect a truss with a curved chord would be a good selection, as, for instance, the parabolic truss or the Schwedler truss where the web members are particularly of light sections. The double intersection Warren truss is unfavorable, as we have seen, but it can be improved by the insertion of verticals, which connect the two single systems in an effective manner.

If in multiple intersection trusses the single trusses act more or less independent of each other, we may predict high secondary stresses, because in such cases the wave like deformations of the chords are rather large.

The secondary stresses in continuous trusses over three supports are very severe at the center and in its neighborhood and for this reason pins at these points appear to be desirable, in order to reduce the stresses.

The width of a member in the plane of the truss should not be greater than buckling and good connections dictate. The transverse width of verticals to which floorbeams are riveted, should also be kept inside proper limits, as otherwise the secondary stresses caused by the floorbeams may prove to be very high. The removal of the material from the axis of the member to its periphery gives the required moment of inertia in the most economical way.

Suspenders should be liberally proportioned to avoid great elongations and consequently large deformations of the bottom chords.

Strict attention should be paid to good detailing of the panel points; the axes of the bars must lie in the plane of the truss and eccentricities should be avoided as far as practicable.

Curved members should not be tolerated under any circum-

stances, and brackets on posts may be used only when nothing better can take their place.

The movable ends of bridges must be kept in proper working order, otherwise they will be the cause of unnecessary stresses.

The use of collision struts cannot be recommended as far as secondary stresses are concerned, because they divide the end posts into two fragments, decreasing the ratio between length and width of these members and increasing the secondary stresses. The collision struts are, as a rule, rather weak members and it is a question whether it would not be better to use the metal on the end posts instead of on these struts, increasing their strength in the direction of the plane of the truss as well as also at right angles to this plane. It is very essential to connect the end posts by a substantial bracing, designing the connections in the best manner possible, and moreover, if a weak end bracing is used the money spent on the horizontal top bracing is wasted.

The position of the horizontal bracings should be selected with a view of reducing eccentricities.

On account of the riveted connections between the floorbeams and the main trusses, the secondary stresses in the vertical posts of the trusses very often exceed by far the limit of stress set by the specifications. Generally speaking this question has not yet been thoroughly settled. A number of suggestions have been made to remedy the defect and also partly carried out. Deep floorbeams tend to reduce the secondary stresses, but they cannot always be made deep enough to be effective from want of depth in the floor, and besides, if they are very deep, they may exceed the limit of economy. The suggestion to give the floorbeams a downward camber, as shown in Fig. 59, originated with Professor Engessor. It is clear that, if a floorbeam is given a deformation corresponding to the load it has to carry and afterwards riveted between the main trusses, it will be quite effective in reducing the secondary stresses in the posts. Another way to avoid these stresses in the verticals would be to give the floorbeams on the main trusses free supports, provided care is taken to properly

transmit the wind and braking forces to the main trusses. An important example to accomplish the object in question is furnished by the double track railroad arch bridge across the Rhine river at the city of Worms, Germany. The river spans consist of two shore spans, each about 351 feet long and one central span of about 388 feet. The main trusses or arch ribs produce vertical reactions only owing to a tension member running along the floor, which intersects with the main trusses. The floor is carried by the main trusses by means of stiff suspenders to which the floorbeams are pin-connected. The design of the floor is such that it is fixed transversely, not by riveting, but by abutting ends, and longitudinally it is fixed exclusively at the center of the span. The main trusses and all of the bracings are riveted work. The intermediate cross bracings are attached to the main trusses by means of plates, in such positions that they offer only a very small resistance to the deformations of the trusses in vertical planes. These arrangements of the details allow a cross-section for unequal loading of the bridge to take the shape of a rhomboid. Here we see then that the reduction of secondary stresses in the suspenders is aimed at by the use of pins and a skillful design of the bracings and their attachments.*

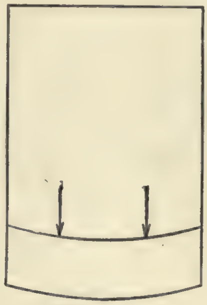


Fig. 59.

* Our practice of stiff connections between floorbeams and main trusses seems to be well founded, as loose-jointed cross constructions may prove to be more injurious to a bridge than severe secondary stresses. But this does not mean that we should leave the secondary stresses to take care of themselves; on the contrary, proper attention should be paid to these with a view to their reduction. Deep floorbeams, substantial gussets and a generous number of rivets to connect the floorbeams to the main trusses are desirable, as also posts, whose width longitudinally and transversely is kept within proper limits.

Although these points are well known to experienced bridge engineers nevertheless the German government called attention to them in printed circulars, issued 1904, for the observance of those engineers who are charged with the designs of engineering structures.

It appears from these circulars that the German government is inclined toward stiff connections between floorbeams and main trusses and that it discourages the designs of hinged floor systems in so far as it requests a proof of their advantages.

It is faulty to design the top chord flanges of the floorbeams exclusively for vertical loads, if at the same time they are also charged with the duty of transmitting the braking forces to the main trusses, which they can only do by being bent in a horizontal plane. In such cases the floorbeam flanges should be provided with brackets, as is shown in Fig. 60, or any other suitable construction may be designed to prevent the bending of these flanges.

We believe the reduction of secondary stresses by the use of

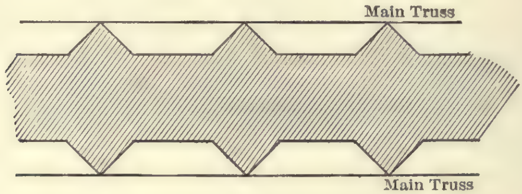


Fig. 60.

pins has been overestimated and that in general the diameters of the pins are too great. Secondary stresses require the pins to be as small in diameter as is consistent with the strength and safety of a bridge. Attention should be paid to have the surfaces, which are in contact with each other, very smooth to facilitate turning.

It has been suggested that light colored paints would be preferable to dark ones, as they absorb less heat and consequently reduce the temperature stresses.

For the reduction of the effects of impact and vibrations the following points deserve consideration. Since the mass of a body which receives a blow must be great if the effects of a blow shall be small, it is therefore beneficial to use heavy floor construction and a heavy track. Full webbed floorbeams and trackstringers and a ballasted track diminish the range of vibrations.

From economical reasons the ties are often placed directly on

the top chords, when the use of a steel floor would be much better.

Tension members should be built stiff, as it is within the range of possibilities that such members, for instance suspenders, are subjected to compression in consequence of vibrations.

Riveted connections and stiff members throughout the bridge are in so far of advantage as they tend to decrease the kinetic energy of vibrations. The riveting of the members at their intersection points, the introduction of secondary and even redundant members, all tend to diminish the time and the amplitude of vibrations.

To further decrease the effects of moving loads, it is necessary to keep the track in perfect working order. The passage from the road bed to the bridge should be smooth. Great attention must be paid to the rail ends at the splices. If these rail ends are not of the same height they will be the cause of severe shocks, and particular stress should be laid on the tightness of the bolts at the rail splices. Long rails are naturally of advantage, since they reduce the number of splices, but on a bridge of short span there should be no rail splice at all.

In reference to derailments and collisions we will mention a few points which are deserving of consideration from the side of the designer. Of course, it cannot be his object to design a bridge so that it is proof against accidents, but he can do a great deal to reduce the effects of accidents.

A ballasted track will prevent the derailed wheels from breaking through the floor. A floor which permits this may cause the collapse of the entire span.

A derailed train at the end of a bridge is likely to strike the end post. Consequently, it is of great importance to provide these posts in through spans, and also the portals, with a very robust constitution.

The use of inner and outer guard rails on bridges, firmly fastened to ties embedded in ballast, is good practice, as also inside guard rails, flaring guards and rerailing frogs on the bridge

approaches. If these guard rails are higher than the track rails and spaced so that the wheels cannot drop into the space between guard rail and track rail, the train runs practically in a groove.

Bridges which are exposed to collisions with floating objects deserve particular attention in their designs. If such collisions are due to causes other than ships, as for example, tree trunks, blocks of ice, etc., it may be sufficient to raise the main trusses above the floorbeams, allowing the latter to take the blows, which would be principally delivered in the direction parallel to their length.

Then the stringers and their connections may be so designed that they are self-supporting, preventing the collapse of the bridge in spite of a disabled main truss.* While the stringers could be charged with this duty for short spans, it is hardly feasible to resort to such means for longer spans.

Drawbridges which are in great danger of coming into collision with ships are likely to have their bottom chord members struck and possibly ruptured so that a collapse would appear to be certain. In such cases it should be the principal aim of the designer to prevent a rupture of a bridge member and this we believe, at least in the case of a bottom chord section, can be done with a high degree of success and at no great expense. Indeed, the extra cost should not play any rôle whatever if all the evil consequences of a collapse of a bridge are considered. The writer would design the stiff web and bottom chord members with the object of giving them increased resisting power against blows, using rather solid plates instead of latticing and lacing, and paying of course great attention to the detailing of the panelpoints. As a further precaution he would brace the bottom chords in a horizontal plane against the track, but without the use of stiff connections. The track could be designed in a manner that it acts like a cushion to dissipate the effects of an impact.

Should the track be above the bottom chord, a collision bracing may extend over the entire width of the bridge, and in this case it

* See *Eng. News*, August 10, 1906.

would take at the same time the functions of the bottom lateral bracing.

If a bridge has sidewalks, so much the better. These can easily be constructed to form an effective protection for the main trusses.

It would be out of place to go here over the manifold details that are possible. It suffices to say that the conditions which govern each individual case must be thoroughly studied in order to properly solve the problem at hand.

Before we conclude we will say a few words in regard to the methods of calculations and their use.

The method of influence lines cannot be used if it is required to consider the effects of deformations of the bars on the result, as in such cases the secondary stresses appear as higher functions of the exterior loads. Where these deformations can be neglected, there is no doubt that the method of influence lines is the best that can be used in so far as it gives the maximum stresses required. But the trouble with this method lies in the fact that it involves an exceedingly great amount of time and labor, which is best appreciated by the one who undertakes an investigation of this sort. This must be the reason why the calculations published so far are nearly all made on the assumption of only one position of the live load.

When the writer undertook the examination of that 140-foot span after Müller-Breslau's method, which is among the examples, he made use of every facility he could think of to shorten his labors. He never went over the same operations the second time unless there was a necessity to do so. He also prepared tables intended to facilitate the overlooking of the situation as much as the nature of the subject allowed him to do. But in spite of all the precautions taken, the computation proved to be a very laborious task indeed. But this is not all. It would be a mistake to believe that that position of the live load which gives the maximum primary stresses is at the same time the one corresponding with the maximum secondary stresses. This may be more or

less true for the chords of a truss resting on two supports, but it is certainly not true for the web members. The writer was not able to foretell either the approximate amount of stress or the deformation of a bar for a given position of the live load. There was only one way to find out something about these points and that was to perform the various operations from beginning to end.

The aspect of this subject changes considerably in case we have to deal with the examination of a truss for just one fixed position of the live load. All methods of calculation of secondary stresses require more time than those for primary stresses, but this should not be a reason to avoid them in cases where the knowledge of these stresses appears to be very desirable. Much depends also on the computer himself in point of time. One man may find it a real hardship to compute the secondary stresses in a 200-foot span for one fixed position of the live load; while another, obtaining his results with ease and rapidity, does not think much of it.

There is no doubt that cases may occur where the safety of the structure imposes the duty on the designer to give an account of the secondary stresses. For instance, old trusses designed for loads which are much lighter than those they actually carry are very good subjects for examinations, because it is here in particular within the range of possibilities that the stresses are raised to a dangerous point. Then there are trusses whose examination appears to be desirable on account of peculiarities in their construction, because they lead us to the expectation of high secondary stresses.

As far as we are aware secondary stresses in trusses of great length, as cantilever trusses, arch ribs, etc., are not known. Such trusses have members whose stresses are subjected to a reversal, which is aggravated by secondary stresses.

The secondary stresses are taken into account by the common practice of lowering the unit stresses, but this is a matter of experience and hardly feasible by theoretical considerations.

Our knowledge of secondary stresses could be improved either

in measuring these stresses by the use of suitable instruments, or by analytical investigations, or by both. The writer suggests that readers who take a particular interest in this subject and have the time to do so, examine trusses and publish their results, which cannot fail to be instructive as they would show us where we have failed.

CHAPTER XI.

LITERATURE.

CONCERNING SECONDARY STRESSES.

Asimont. Hauptspannung und Sekundärspannung. Zeitschrift für Baukunde, 1880.

Manderla. Die Berechnung der Sekundärspannungen, welche im einfachen Fachwerke infolge starrer Knotenverbindungen entstehen. Allgemeine Bauzeitung, 1880, p. 34.

Jebens. Die Spannungen in den Vertikalständern der eisernen Brücken. Zeitschrift des Vereins deutscher Ingenieure, 1880, p. 127.

Manderla. Formänderung des Fachwerks bei wechselnder Belastung. Allgemeine Bauzeitung, 1884, p. 81, 89.

Engesser. Die Sicherung offener Brücken gegen Ausknicken. Centralblatt der Bauverwaltung, 1884, p. 415; 1885, p. 71.

Ritter. Über die Druckfestigkeit stabförmiger Körper mit besonderer Rücksicht auf die im steifen Fachwerk auftretenden Nebenspannungen. Schweizerische Bauzeitung, 1884, I, p. 37, 43, 47.

Müller-Breslau. Über Biegungsspannungen in Fachwerken. Allgemeine Bauzeitung, 1885, p. 85, 89.

Landsberg. Ebene Fachwerkssysteme mit festen Knotenpunkten und das Princip der Deformationsarbeit. Centralblatt der Bauverwaltung, 1885, p. 165.

Landsberg. Beitrag zur Theorie der Fachwerke (graphische Ermittelung der Sekundärspannungen infolge fester Knotenverbindungen der Gurtstäbe). Zeitschrift des Architekten-und Ingenieur-Vereins zu Hannover, 1885, p. 361.

Müller-Breslau. Beitrag zur Theorie des Fachwerks. Zeitschrift des Architekten-und Ingenieur-Vereins zu Hannover, 1885, p. 417.

Weyrauch. Aufgaben zur Theorie elastischer Körper. Leipzig, 1885, p. 269.

Manderla. Über die Wirkungsweise gelenkförmiger Knotenverbindungen. Allgemeine Bauzeitung, 1886, p. 9, 20, 32, 37.

Landsberg. Beitrag zur Theorie der Fachwerke. Zeitschrift des Architekten- und Ingenieur-Vereins zu Hannover, 1886, p. 195.

Müller-Breslau. Zur Theorie der Biegungsspannungen in Fachwerkträgern. Zeitschrift des Architekten-und Ingenieur-Vereins zu Hannover, 1886, p. 399.

Winkler. Äussere Kräfte gerader Träger, 1886, p. 166 and 1875, p. 169, 170.

Winkler. Querkonstruktionen, p. 179–182.

Landsberg. Beitrag zur Theorie des ebenen Fachwerks. Festschrift der technischen Hochschule zu Darmstadt, 1886.

Considère. Note sur les effets anormaux dans les ouvrages metalliques. Annales des ponts et chaussées, 1887, I, p. 372.

Fränkel und Krüger. Spannungs-und Formänderungsmessungen an dem eisernen Pendelpfeiler Viadukte über das Oschützthal bei Weida. Civil Ingenieur 1887, p. 439. Nebenspannungen der Pfeiler, p. 484.

Allievi. Equilibrio internio delle pile metalliche. Roma, 1882. (Translated into German by Totz, Wien, 1888.)

Hacker. Über Biegungspannungen in Schwedler'schen Kuppeln. Zeitschrift des Architekten-und Ingenieur-Vereins zu Hannover, 1888, p. 223.

Müller-Breslau. Beitrag zur Theorie der ebenen elastischen Träger. Zeitschrift des Architekten-und Ingenieur-Vereins zu Hannover, 1888, p. 605.

Ritter. Anwendungen der graphischen Statik. II, Das Fachwerk. Zürich, 1890, p. 171.

Handbuch der Ingenieurwissenschaften. Vol. II. 1890.

Brick. Fachwissenschaftliche Erörterungen zu dem Berichte des Brückenmaterialkomites über die durchgeführten Versuche mit genieteten Trägern aus Flusseisen und Schweisseisen. Zeitschrift des Österreichischen Ingenieur-und Architekten-Vereins, 1891, p. 76.

Jebens. Die seitliche Standsicherheit von eisernen Brücken ohne oberen Querverband. Centralblatt der Bauverwaltung, 1892, p. 148.

Engesser. Die seitliche Standfestigkeit offener Brücken. Centralblatt der Bauverwaltung, 1892, p. 349.

Engesser. Die Zusatzkräfte und Nebenspannungen eiserner Fachwerkbrücken. I, Die Zusatzkräfte. Berlin, 1892. II, Die Nebenspannungen. Berlin, 1893.

Barkhausen. Der Steifrahmen im Wind-und Querverbande geschlossener Trogbrücken. Zeitschrift des Vereins deutscher Ingenieure, 1892, p. 421, 492.

Barkhausen. Biegungsspannungen in Blechen und Bändern infolge von einseitiger Verlaschung oder von Überlappungsverbindungen. Zeitschrift des Vereins deutscher Ingenieure, 1892, p. 553.

Mohr. Die Berechnung der Fachwerke mit starren Knotenverbindungen. Der Civil Ingenieur: Organ des Sächsischen Ingenieur-und Architekten-Vereins, 1892, p. 577; 1893, p. 67.

Jaquier. Note sur les efforts secondaires qui peuvent se produire dans les systèmes articulés à attaches rigides. Annales des ponts et chaussées, 1893, I, p. 1142.

Engesser. Die zusätzlichen Beanspruchungen durchgehender (kontinuirlicher) Brückenkonstruktionen. Zeitschrift für Bauwesen, 1894, p. 305.

Engesser. Über die Verringerung der Nebenspannungen von Fachwerkbrücken durch die Art der Aufstellung. Centralblatt der Bauverwaltung, 1895, p. 317.

Rapport sur les épreuves de charge jusqu' à rupture de l'ancien pont sur l'Emme à Wolhusen. Berne, 1895.

Häseler. Berechnung der auf Verdrehung beanspruchten Brückenträger. Zeitschrift des Vereins deutscher Ingenieure, 1896, p. 761.

Dupuy. Résistances des barres soumises à des efforts agissant parallèlement à leur axe neutre et en dehors de cette axe. Annales des ponts et chaussées, 1896, II, p. 223.

Häseler. Der Brückenbau. I, Die eisernen Brücken. 3. Lief. Braunschweig, 1897.

Luegers. Lexikon der gesammten Technik mit ihren Hülfswissenschaften im Verein mit Fachgenossen herausgegeben.

Franke. Berechnung der Durchbiegung und der Nebenspannungen der Fachwerkträger. Zeitschrift für Bauwesen, 1898.

Patton. Beitrag zur Berechnung der Nebenspannungen infolge starrer Knotenverbindungen bei Brückenträgern. Zeitschrift für Architektur-and Ingenieurwesen. Heft 4, 1902.

Isami Hiroi. Statically Indeterminate Bridge Stresses. 1905.

Mehrtens. Vorlesungen über Statik der Baukonstruktionen und Festigkeitslehre. Vol. III. 1905.

Mohr. Abhandlungen aus dem Gebicte der technischen Mechanik. 1906.

CONCERNING IMPACT AND VIBRATIONS.

Résal. Effet des charges roulantes. Annales des ponts et chaussées, 1882, II, p. 337–352.

Résal. Effet des charges roulantes sur les ponts métalliques. Annales des ponts et chaussées, 1883, I, p. 277–299.

Robinson. Vibration of bridges. Transactions of the American Society of Civil Engneers, 1887, Vol. XVI.

Soulyere. Action dynamiques des charges roulantes sur les poutres rigides qui ne travaillent qu' a la flexion. Annales des ponts et chaussées, 1889, p. 341–441.

Glauser. Dynamische Wirkungen bewegter Lasten auf eiserne Brücken. Glaser's Annalen für Gewerbe und Bauwesen, 1891, Vol. 29, p. 113; 1892, Vol. 30, p. 61; 1894, Vol. 34, p. 56.

Zimmermann. Die Wirkungen bewegter Lasten auf Brücken. Centralblatt der Bauverwaltung, 1891, p. 448; 1892, p. 159, 199, 215.

Melan. Über die dynamische Wirkung bewegter Lasten auf Brücken. Zeitschrift des österreichischen Ingenieur-und Architekten-Vereins, 1893, p. 293.

Melan. Über die dynamische Wirkung bewegter Lasten auf eiserne Brücken. Glaser's Annalen für Gewerbe und Bauwesen, 1894.

Deslandres. Note sur les épreuves par charge roulante et l'action des chocs. Annales des ponts et chaussées, 1894, I, p. 735.

Zimmermann. Die Schwingungen eines Trägers mit bewegter Last. Berlin, 1896.

Stone. The determination of the safe working stress for railway bridges of wrought iron and steel. Transactions of the American Society of Civil Engineers, 1889. Vol. XLI.

Turneaure. Some experiments on bridges under moving train-loads. Transactions of the American Society of Civil Engineers, 1899, Vol. XLI.

SHORT-TITLE CATALOGUE

OF THE

PUBLICATIONS

OF

JOHN WILEY & SONS,

NEW YORK.

LONDON: CHAPMAN & HALL, LIMITED.

ARRANGED UNDER SUBJECTS.

Descriptive circulars sent on application. Books marked with an asterisk (*) are sold at *net* prices only. All books are bound in cloth unless otherwise stated.

AGRICULTURE.

Armsby's Manual of Cattle-feeding	12mo,	$1 75
Principles of Animal Nutrition	8vo,	4 00
Budd and Hansen's American Horticultural Manual:		
Part I. Propagation, Culture, and Improvement	12mo,	1 50
Part II. Systematic Pomology	12mo,	1 50
Elliott's Engineering for Land Drainage	12mo,	1 50
Practical Farm Drainage	12mo,	1 00
Graves's Forest Mensuration	8vo,	4 00
Green's Principles of American Forestry	12mo,	1 50
Grotenfelt's Principles of Modern Dairy Practice. (Woll.)	12mo,	2 00
Hanausek's Microscopy of Technical Products. (Winton.)	8vo,	5 00
Herrick's Denatured or Industrial Alcohol	8vo,	4 00
Maynard's Landscape Gardening as Applied to Home Decoration	12mo,	1 50
* McKay and Larsen's Principles and Practice of Butter-making	8vo,	1 50
Sanderson's Insects Injurious to Staple Crops	12mo,	1 50
* Schwarz's Longleaf Pine in Virgin Forest	12mo,	1 25
Stockbridge's Rocks and Soils	8vo,	2 50
Winton's Microscopy of Vegetable Foods	8vo,	7 50
Woll's Handbook for Farmers and Dairymen	16mo,	1 50

ARCHITECTURE.

Baldwin's Steam Heating for Buildings	12mo,	2 50
Bashore's Sanitation of a Country House	12mo,	1 00
Berg's Buildings and Structures of American Railroads	4to,	5 00
Birkmire's Planning and Construction of American Theatres	8vo,	3 00
Architectural Iron and Steel	8vo,	3 50
Compound Riveted Girders as Applied in Buildings	8vo,	2 00
Planning and Construction of High Office Buildings	8vo,	3 50
Skeleton Construction in Buildings	8vo,	3 00
Brigg's Modern American School Buildings	8vo,	4 00
Carpenter's Heating and Ventilating of Buildings	8vo,	4 00

Freitag's Architectural Engineering. 8vo. 3 50
Fireproofing of Steel Buildings. 8vo, 2 50
French and Ives's Stereotomy. 8vo, 2 50
Gerhard's Guide to Sanitary House-inspection. 16mo, 1 00
Sanitation of Public Buildings 12mo, 1 50
Theatre Fires and Panics. 12mo, 1 50
*Greene's Structural Mechanics 8vo, 2 50
Holly's Carpenters' and Joiners' Handbook. 18mo, 75
Johnson's Statics by Algebraic and Graphic Methods. 8vo, 2 00
Kellaway's How to Lay Out Suburban Home Grounds. 8vo, 2 00
Kidder's Architects' and Builders' Pocket-book. Rewritten Edition. 16mo, mor., 5 00
Merrill's Stones for Building and Decoration. 8vo, 5 00
Non-metallic Minerals: Their Occurrence and Uses. 8vo, 4 00
Monckton's Stair-building. 4to, 4 00
Patton's Practical Treatise on Foundations. 8vo, 5 00
Peabody's Naval Architecture. 8vo, 7 50
Rice's Concrete-block Manufacture 8vo, 2 00
Richey's Handbook for Superintendents of Construction. . . . . .16mo, mor., 4 00
* Building Mechanics' Ready Reference Book:
* Carpenters' and Woodworkers' Edition. . . . . . . .16mo, morocco, 1 50
* Cementworkers and Plasterer's Edition. (In Press.)
* Stone- and Brick-mason's Edition. . . . . .12mo, mor., 1 50
Sabin's Industrial and Artistic Technology of Paints and Varnish. 8vo, 3 00
Siebert and Biggin's Modern Stone-cutting and Masonry. 8vo, 1 50
Snow's Principal Species of Wood. 8vo, 3 50
Sondericker's Graphic Statics with Applications to Trusses, Beams, and Arches.
8vo, 2 00
Towne's Locks and Builders' Hardware. . . . . .18mo, morocco, 3 00
Turneaure and Maurer's Principles of Reinforced Concrete Construction. 8vo, 3 00
Wait's Engineering and Architectural Jurisprudence 8vo, 6 00
Sheep, 6 50
Law of Operations Preliminary to Construction in Engineering and Architecture. 8vo, 5 00
Sheep, 5 50
Law of Contracts. 8vo, 3 00
Wilson's Air Conditioning, (In Press.)
Wood's Rustless Coatings: Corrosion and Electrolysis of Iron and Steel. .8vo, 4 00
Worcester and Atkinson's Small Hospitals, Establishment and Maintenance, Suggestions for Hospital Architecture, with Plans for a Small Hospital.
12mo, 1 25
The World's Columbian Exposition of 1893. .Large 4to, 1 00

ARMY AND NAVY.

Bernadou's Smokeless Powder, Nitro-cellulose, and the Theory of the Cellulose Molecule. 12mo, 2 50
Chase's Screw Propellers and Marine Propulsion. 8vo, 3 00
Cloke's Gunner's Examiner. 8vo, 1 50
Craig's Azimuth. 4to, 3 50
Crehore and Squier's Polarizing Photo-chronograph. 8vo, 3 00
* Davis's Elements of Law. 8vo, 2 50
* Treatise on the Military Law of United States. 8vo, 7 00
Sheep, 7 50
De Brack's Cavalry Outposts Duties. (Carr.). . . . . .24mo, morocco, 2 00
Dietz's Soldier's First Aid Handbook. . . . . .16mo, morocco, 1 25
* Dudley's Military Law and the Procedure of Courts-martial. . . Large 12mo, 2 50
Durand's Resistance and Propulsion of Ships. 8vo, 5 00

* Dyer's Handbook of Light Artillery. 12mo, 3 00
Eissler's Modern High Explosives. 8vo, 4 00
* Fiebeger's Text-book on Field Fortification. . . . . Small 8vo, 2 00
Hamilton's The Gunner's Catechism 18mo, 1 00
* Hoff's Elementary Naval Tactics. 8vo, 1 50
Ingalls's Handbook of Problems in Direct Fire. 8vo, 4 00
* Lissak's Ordnance and Gunnery 8vo, 6 00
* Lyons's Treatise on Electromagnetic Phenomena. Vols. I. and II. . 8vo, each, 6 00
* Mahan's Permanent Fortifications. (Mercur.). . . . . 8vo, half morocco, 7 50
Manual for Courts-martial. . . . . 16mo, morocco, 1 50
* Mercur's Attack of Fortified Places. 12mo, 2 00
* Elements of the Art of War. 8vo, 4 00
Metcalf's Cost of Manufactures—And the Administration of Workshops. . 8vo, 5 00
* Ordnance and Gunnery. 2 vols. 12mo, 5 00
Murray's Infantry Drill Regulations. . . . . 18mo, paper, 10
Nixon's Adjutants' Manual. 24mo, 1 00
Peabody's Naval Architecture. 8vo, 7 50
* Phelps's Practical Marine Surveying. 8vo, 2 50
Powell's Army Officer's Examiner. 12mo, 4 00
Sharpe's Art of Subsisting Armies in War. 18mo, morocco, 1 50
* Tupes and Poole's Manual of Bayonet Exercises and Musketry Fencing.
24mo, leather, 50
Weaver's Military Explosives. 8vo, 3 00
Wheeler's Siege Operations and Military Mining. 8vo, 2 00
Winthrop's Abridgment of Military Law. 12mo, 2 50
Woodhull's Notes on Military Hygiene. 16mo, 1 50
Young's Simple Elements of Navigation. 16mo, morocco, 2 00

ASSAYING.

Fletcher's Practical Instructions in Quantitative Assaying with the Blowpipe.
12mo, morocco, 1 50
Furman's Manual of Practical Assaying. 8vo, 3 00
Lodge's Notes on Assaying and Metallurgical Laboratory Experiments. . . . 8vo, 3 00
Low's Technical Methods of Ore Analysis. 8vo, 3 00
Miller's Manual of Assaying. 12mo, 1 00
Cyanide Process. 12mo, 1 00
Minet's Production of Aluminum and its Industrial Use. (Waldo.). 12mo, 2 50
O'Driscoll's Notes on the Treatment of Gold Ores. 8vo, 2 00
Ricketts and Miller's Notes on Assaying. 8vo, 3 00
Robine and Lenglen's Cyanide Industry. (Le Clerc.). 8vo, 4 00
Ulke's Modern Electrolytic Copper Refining. 8vo, 3 00
Wilson's Cyanide Processes. 12mo, 1 50
Chlorination Process. 12mo, 1 50

ASTRONOMY.

Comstock's Field Astronomy for Engineers. 8vo, 2 50
Craig's Azimuth. 4to, 3 50
Crandall's Text-book on Geodesy and Least Squares. 8vo, 3 00
Doolittle's Treatise on Practical Astronomy. 8vo, 4 00
Gore's Elements of Geodesy. 8vo, 2 50
Hayford's Text-book of Geodetic Astronomy. 8vo, 3 00
Merriman's Elements of Precise Surveying and Geodesy. 8vo, 2 50
* Michie and Harlow's Practical Astronomy. 8vo, 3 00
* White's Elements of Theoretical and Descriptive Astronomy 12mo, 2 00

BOTANY.

Davenport's Statistical Methods, with Special Reference to Biological Variation. 16mo, morocco, 1 25
Thomé and Bennett's Structural and Physiological Botany............16mo, 2 25
Westermaier's Compendium of General Botany. (Schneider.)..........8vo, 2 00

CHEMISTRY.

* Abegg's Theory of Electrolytic Dissociation. (Von Ende.)..........12mo, 1 25
Adriance's Laboratory Calculations and Specific Gravity Tables........12mo, 1 25
Alexeyeff's General Principles of Organic Synthesis. (Matthews.)........8vo, 3 00
Allen's Tables for Iron Analysis..................................8vo, 3 00
Arnold's Compendium of Chemistry. (Mandel.)...............Small 8vo, 3 50
Austen's Notes for Chemical Students..............................12mo, 1 50
Beard's Mine Gases and Explosions. (In Press.)
Bernadou's Smokeless Powder.—Nitro-cellulose, and Theory of the Cellulose Molecule..12mo, 2 50
Bolduan's Immune Sera..12mo, 1 50
* Browning's Introduction to the Rarer Elements....................8vo, 1 50
Brush and Penfield's Manual of Determinative Mineralogy.............8vo, 4 00
* Claassen's Beet-sugar Manufacture. (Hall and Rolfe.)...............8vo, 3 00
Classen's Quantitative Chemical Analysis by Electrolysis. (Boltwood.)..8vo, 3 00
Cohn's Indicators and Test-papers................................12mo, 2 00
Tests and Reagents...8vo, 3 00
Crafts's Short Course in Qualitative Chemical Analysis. (Schaeffer.)...12mo, 1 50
* Danneel's Electrochemistry. (Merriam.)..........................12mo, 1 25
Dolezalek's Theory of the Lead Accumulator (Storage Battery). (Von Ende.)...12mo, 2 50
Drechsel's Chemical Reactions. (Merrill.)........................12mo, 1 25
Duhem's Thermodynamics and Chemistry. (Burgess.)................8vo, 4 00
Eissler's Modern High Explosives..................................8vo, 4 00
Effront's Enzymes and their Applications. (Prescott.)................8vo, 3 00
Erdmann's Introduction to Chemical Preparations. (Dunlap.).......12mo, 1 25
* Fischer's Physiology of Alimentation.....................Large 12mo, 2 00
Fletcher's Practical Instructions in Quantitative Assaying with the Blowpipe. 12mo, morocco, 1 50
Fowler's Sewage Works Analyses..................................12mo, 2 00
Fresenius's Manual of Qualitative Chemical Analysis. (Wells.).........8vo, 5 00
Manual of Qualitative Chemical Analysis. Part I. Descriptive. (Wells.) 8vo, 3 00
Quantitative Chemical Analysis. (Cohn.) 2 vols.................8vo, 12 50
Fuertes's Water and Public Health................................12mo, 1 50
Furman's Manual of Practical Assaying............................8vo, 3 00
* Getman's Exercises in Physical Chemistry........................12mo, 2 00
Gill's Gas and Fuel Analysis for Engineers........................12mo, 1 25
* Gooch and Browning's Outlines of Qualitative Chemical Analysis. Small 8vo, 1 25
Grotenfelt's Principles of Modern Dairy Practice. (Woll.)............12mo, 2 00
Groth's Introduction to Chemical Crystallography (Marshall)........12mo, 1 25
Hammarsten's Text-book of Physiological Chemistry. (Mandel.).......8vo, 4 00
Hanausek's Microscopy of Technical Products. (Winton.)...............8vo, 5 00
* Haskin's and MacLeod's Organic Chemistry.......................12mo, 2 00
Helm's Principles of Mathematical Chemistry. (Morgan.)..............12mo, 1 50
Hering's Ready Reference Tables (Conversion Factors).....16mo, morocco, 2 50
Herrick's Denatured or Industrial Alcohol...........................8vo, 4 00
Hind's Inorganic Chemistry.......................................8vo, 3 00
* Laboratory Manual for Students..............................12mo, 1 00
Holleman's Text-book of Inorganic Chemistry. (Cooper.)..............8vo, 2 50
Text-book of Organic Chemistry. (Walker and Mott.)...............8vo, 2 50
* Laboratory Manual of Organic Chemistry. (Walker.)...........12mo, 1 00

Holley and Ladd's Analysis of Mixed Paints, Color Pigments, and Varnishes. (In Press)
Hopkins's Oil-chemists' Handbook 8vo, 3 00
Iddings's Rock Minerals 8vo, 5 00
Jackson's Directions for Laboratory Work in Physiological Chemistry .. 8vo, 1 25
Johannsen's Key for the Determination of Rock-forming Minerals in Thin Sections. (In Press)
Keep's Cast Iron 8vo, 2 50
Ladd's Manual of Quantitative Chemical Analysis 12mo, 1 00
Landauer's Spectrum Analysis. (Tingle.) 8vo, 3 00
* Langworthy and Austen. The Occurrence of Aluminium in Vegetable Products, Animal Products, and Natural Waters 8vo, 2 00
Lassar-Cohn's Application of Some General Reactions to Investigations in Organic Chemistry. (Tingle.) 12mo, 1 00
Leach's The Inspection and Analysis of Food with Special Reference to State Control 8vo, 7 50
Löb's Electrochemistry of Organic Compounds. (Lorenz.) 8vo, 3 00
Lodge's Notes on Assaying and Metallurgical Laboratory Experiments 8vo, 3 00
Low's Technical Method of Ore Analysis 8vo, 3 00
Lunge's Techno-chemical Analysis. (Cohn.) 12mo 1 00
* McKay and Larsen's Principles and Practice of Butter-making 8vo, 1 50
Maire's Modern Pigments and their Vehicles. (In Press.)
Mandel's Handbook for Bio-chemical Laboratory 12mo, 1 50
* Martin's Laboratory Guide to Qualitative Analysis with the Blowpipe .. 12mo, 60
Mason's Water-supply. (Considered Principally from a Sanitary Standpoint.) 3d Edition, Rewritten 8vo, 4 00
Examination of Water. (Chemical and Bacteriological.) 12mo, 1 25
Matthew's The Textile Fibres. 2d Edition, Rewritten 8vo, 4 00
Meyer's Determination of Radicles in Carbon Compounds. (Tingle.) .. 12mo, 1 00
Miller's Manual of Assaying 12mo, 1 00
Cyanide Process 12mo, 1 00
Minet's Production of Aluminum and its Industrial Use. (Waldo.) 12mo, 2 50
Mixter's Elementary Text-book of Chemistry 12mo, 1 50
Morgan's An Outline of the Theory of Solutions and its Results 12mo, 1 00
Elements of Physical Chemistry 12mo, 3 00
* Physical Chemistry for Electrical Engineers 12mo, 5 00
Morse's Calculations used in Cane-sugar Factories .... 16mo, morocco, 1 50
* Muir's History of Chemical Theories and Laws 8vo, 4 00
Mulliken's General Method for the Identification of Pure Organic Compounds. Vol. I .... Large 8vo, 5 00
O'Driscoll's Notes on the Treatment of Gold Ores 8vo, 2 00
Ostwald's Conversations on Chemistry. Part One. (Ramsey.) 12mo, 1 50
" " " " Part Two. (Turnbull.) 12mo, 2 00
* Palmer's Practical Test Book of Chemistry 12mo, 1 00
* Pauli's Physical Chemistry in the Service of Medicine. (Fischer.) 12mo, 1 25
* Penfield's Notes on Determinative Mineralogy and Record of Mineral Tests. 8vo, paper, 50
Pictet's The Alkaloids and their Chemical Constitution. (Biddle.) 8vo, 5 00
Pinner's Introduction to Organic Chemistry. (Austen.) 12mo, 1 50
Poole's Calorific Power of Fuels 8vo, 3 00
Prescott and Winslow's Elements of Water Bacteriology, with Special Reference to Sanitary Water Analysis 12mo, 1 25
* Reisig's Guide to Piece-dyeing 8vo, 25 00
Richards and Woodman's Air, Water, and Food from a Sanitary Standpoint .. 8vo, 2 00
Ricketts and Miller's Notes on Assaying 8vo, 3 00
Rideal's Sewage and the Bacterial Purification of Sewage 8vo, 4 00
Disinfection and the Preservation of Food 8vo, 4 00
Riggs's Elementary Manual for the Chemical Laboratory 8vo, 1 25
Robine and Lenglen's Cyanide Industry. (Le Clerc.) 8vo, 4 00

Ruddiman's Incompatibilities in Prescriptions. 8vo, 2 00
* Whys in Pharmacy 12mo, 1 00
Sabin's Industrial and Artistic Technology of Paints and Varnish. 8vo, 3 00
Salkowski's Physiological and Pathological Chemistry. (Orndorff.) 8vo, 2 50
Schimpf's Text-book of Volumetric Analysis. 12mo, 2 50
Essentials of Volumetric Analysis. 12mo, 1 25
* Qualitative Chemical Analysis. 8vo, 1 25
Smith's Lecture Notes on Chemistry for Dental Students. 8vo, 2 50
Spencer's Handbook for Chemists of Beet-sugar Houses. 16mo, morocco 3 00
Handbook for Cane Sugar Manufacturers. 16mo, morocco, 3 00
Stockbridge's Rocks and Soils. 8vo, 2 50
* Tillman's Elementary Lessons in Heat. 8vo, 1 50
* Descriptive General Chemistry. 8vo, 3 00
Treadwell's Qualitative Analysis. (Hall.) 8vo, 3 00
Quantitative Analysis. (Hall.) 8vo, 4 00
Turneaure and Russell's Public Water-supplies. 8vo, 5 00
Van Deventer's Physical Chemistry for Beginners. (Boltwood.) 12mo, 1 50
* Walke's Lectures on Explosives. 8vo, 4 00
Ware's Beet-sugar Manufacture and Refining. Vol. I. Small 8vo, 4 00
" " " " " Vol. II. Small 8vo, 5 00
Washington's Manual of the Chemical Analysis of Rocks. 8vo, 2 00
Weaver's Military Explosives. 8vo, 3 00
Wehrenfennig's Analysis and Softening of Boiler Feed-Water 8vo, 4 00
Wells's Laboratory Guide in Qualitative Chemical Analysis. 8vo, 1 50
Short Course in Inorganic Qualitative Chemical Analysis for Engineering Students. 12mo, 1 50
Text-book of Chemical Arithmetic 12mo, 1 25
Whipple's Microscopy of Drinking-water. 8vo, 3 50
Wilson's Cyanide Processes 12mo, 1 50
Chlorination Process. 12mo, 1 50
Winton's Microscopy of Vegetable Foods 8vo, 7 50
Wulling's Elementary Course in Inorganic, Pharmaceutical, and Medical Chemistry. 12mo, 2 00

CIVIL ENGINEERING.

BRIDGES AND ROOFS. HYDRAULICS. MATERIALS OF ENGINEERING RAILWAY ENGINEERING.

Baker's Engineers' Surveying Instruments. 12mo, 3 00
Bixby's Graphical Computing Table. Paper 19½ × 24¼ inches. 25
Breed and Hosmer's Principles and Practice of Surveying. 8vo, 3 00
* Burr's Ancient and Modern Engineering and the Isthmian Canal 8vo, 3 50
Comstock's Field Astronomy for Engineers. 8vo, 2 50
* Corthell's Allowable Pressures on Deep Foundations. 12mo, 1 25
Crandall's Text-book on Geodesy and Least Squares 8vo, 3 00
Davis's Elevation and Stadia Tables. 8vo, 1 00
Elliott's Engineering for Land Drainage. 12mo, 1 50
Practical Farm Drainage. 12mo, 1 00
*Fiebeger's Treatise on Civil Engineering. 8vo, 5 00
Flemer's Phototopographic Methods and Instruments. 8vo, 5 00
Folwell's Sewerage. (Designing and Maintenance.) 8vo, 3 00
Freitag's Architectural Engineering. 2d Edition, Rewritten 8vo, 3 50
French and Ives's Stereotomy. 8vo, 2 50
Goodhue's Municipal Improvements. 12mo, 1 50
Gore's Elements of Geodesy. 8vo, 2 50
* Hauch and Rice's Tables of Quantities for Preliminary Estimates. 12mo, 1 25
Hayford's Text-book of Geodetic Astronomy. 8vo, 3 00

Hering's Ready Reference Tables (Conversion Factors). . . . . .16mo, morocco, 2 50
Howe's Retaining Walls for Earth. 12mo, 1 25
Hoyt and Grover's River Discharge. 8vo, 2 00
* Ives's Adjustments of the Engineer's Transit and Level. . . . . .16mo, Bds. 25
Ives and Hilts's Problems in Surveying . . . . .16mo, morocco, 1 50
Johnson's (J. B.) Theory and Practice of Surveying. . . . . .Small 8vo, 4 00
Johnson's (L. J.) Statics by Algebraic and Graphic Methods. 8vo, 2 00
Laplace's Philosophical Essay on Probabilities. (Truscott and Emory.). 12mo, 2 00
Mahan's Treatise on Civil Engineering. (1873.) (Wood.). 8vo, 5 00
* Descriptive Geometry. 8vo, 1 50
Merriman's Elements of Precise Surveying and Geodesy. 8vo, 2 50
Merriman and Brooks's Handbook for Surveyors. . . . . .16mo, morocco, 2 00
Nugent's Plane Surveying. 8vo, 3 50
Ogden's Sewer Design. 12mo, 2 00
Parsons's Disposal of Municipal Refuse. 8vo, 2 00
Patton's Treatise on Civil Engineering. .8vo half leather, 7 50
Reed's Topographical Drawing and Sketching 4to, 5 00
Rideal's Sewage and the Bacterial Purification of Sewage. 8vo, 4 00
Riemer's Shaft-sinking under Difficult Conditions. (Corning and Peele.) . .8vo, 3 00
Siebert and Biggin's Modern Stone-cutting and Masonry. 8vo, 1 50
Smith's Manual of Topographical Drawing. (McMillan.). 8vo, 2 50
Sondericker's Graphic Statics, with Applications to Trusses, Beams, and Arches.
8vo, 2 00
Taylor and Thompson's Treatise on Concrete, Plain and Reinforced. 8vo, 5 00
Tracy's Plane Surveying. . . . . 16mo, morocco, 3 00
* Trautwine's Civil Engineer's Pocket-book. . . . . .16mo, morocco, 5 00
Venable's Garbage Crematories in America. 8vo, 2 00
Wait's Engineering and Architectural Jurisprudence. 8vo, 6 00
Sheep, 6 50
Law of Operations Preliminary to Construction in Engineering and Architecture. 8vo, 5 00
Sheep, 5 50
Law of Contracts. 8vo, 3 00
Warren's Stereotomy—Problems in Stone-cutting. .8vo, 2 50
Webb's Problems in the Use and Adjustment of Engineering Instruments.
16mo, morocco, 1 25
Wilson's Topographic Surveying. 8vo, 3 50

BRIDGES AND ROOFS.

Boller's Practical Treatise on the Construction of Iron Highway Bridges. .8vo, 2 00
Burr and Falk's Influence Lines for Bridge and Roof Computations. 8vo, 3 00
Design and Construction of Metallic Bridges 8vo, 5 00
Du Bois's Mechanics of Engineering. Vol. II. . . . . .Small 4to, 10 00
Foster's Treatise on Wooden Trestle Bridges. 4to, 5 00
Fowler's Ordinary Foundations 8vo, 3 50
Greene's Roof Trusses. 8vo, 1 25
Bridge Trusses. 8vo, 2 50
Arches in Wood, Iron, and Stone. 8vo, 2 50
Grimm's Secondary Stresses in Bridge Trusses. (In Press.)
Howe's Treatise on Arches. 8vo, 4 00
Design of Simple Roof-trusses in Wood and Steel. 8vo, 2 00
Symmetrical Masonry Arches. 8vo, 2 50
Johnson, Bryan, and Turneaure's Theory and Practice in the Designing of Modern Framed Structures. . . . . .Small 4to, 10 00
Merriman and Jacoby's Text-book on Roofs and Bridges:
Part I. Stresses in Simple Trusses. 8vo, 2 50
Part II. Graphic Statics. 8vo, 2 50
Part III. Bridge Design. 8vo, 2 50
Part IV. Higher Structures 8vo, 2 50

Morison's Memphis Bridge. 4to, 10 00
Waddell's De Pontibus, a Pocket-book for Bridge Engineers. .16mo, morocco, 2 00
* Specifications for Steel Bridges. 12mo, 50
Wright's Designing of Draw-spans. Two parts in one volume..........8vo, 3 50

HYDRAULICS.

Barnes's Ice Formation. 8vo, 3 00
Bazin's Experiments upon the Contraction of the Liquid Vein Issuing from an Orifice. (Trautwine.)...................................8vo, 2 00
Bovey's Treatise on Hydraulics.......................................8vo, 5 00
Church's Mechanics of Engineering....................................8vo, 6 00
Diagrams of Mean Velocity of Water in Open Channels..........paper, 1 50
Hydraulic Motors. 8vo, 2 00
Coffin's Graphical Solution of Hydraulic Problems. ....16mo, morocco, 2 50
Flather's Dynamometers, and the Measurement of Power. 12mo, 3 00
Folwell's Water-supply Engineering. 8vo, 4 00
Frizell's Water-power...8vo, 5 00
Fuertes's Water and Public Health. 12mo, 1 50
Water-filtration Works. 12mo, 2 50
Ganguillet and Kutter's General Formula for the Uniform Flow of Water in Rivers and Other Channels. (Hering and Trautwine.). 8vo, 4 00
Hazen's Clean Water and How to Get It......................Large 12mo, 1 50
Filtration of Public Water-supply...................................8vo, 3 00
Hazlehurst's Towers and Tanks for Water-works. 8vo, 2 50
Herschel's 115 Experiments on the Carrying Capacity of Large, Riveted, Metal Conduits. 8vo, 2 00
* Hubbard and Kiersted's Water-works Management and Maintenance. . 8vo, 4 00
Mason's Water-supply. (Considered Principally from a Sanitary Standpoint.) 8vo, 4 00
Merriman's Treatise on Hydraulics. 8vo, 5 00
* Michie's Elements of Analytical Mechanics. 8vo, 4 00
Schuyler's Reservoirs for Irrigation, Water-power, and Domestic Water-supply. ....Large 8vo, 5 00
* Thomas and Watt's Improvement of Rivers.......................... 4to, 6 00
Turneaure and Russell's Public Water-supplies 8vo, 5 00
Wegmann's Design and Construction of Dams. 5th Edition, enlarged . . .4to, 6 00
Water-supply of the City of New York from 1658 to 1895. ....4to, 10 00
Whipple's Value of Pure Water...........................Large 12mo, 1 00
Williams and Hazen's Hydraulic Tables. 8vo, 1 50
Wilson's Irrigation Engineering. ....Small 8vo, 4 00
Wolff's Windmill as a Prime Mover.....................................8vo, 3 00
Wood's Turbines. 8vo, 2 50
Elements of Analytical Mechanics. 8vo, 3 00

MATERIALS OF ENGINEERING.

Baker's Treatise on Masonry Construction.............................8vo, 5 00
Roads and Pavements. 8vo, 5 00
Black's United States Public Works ....Oblong 4to, 5 00
* Bovey's Strength of Materials and Theory of Structures. 8vo, 7 50
Burr's Elasticity and Resistance of the Materials of Engineering. 8vo, 7 50
Byrne's Highway Construction. 8vo, 5 00
Inspection of the Materials and Workmanship Employed in Construction. 16mo, 3 00
Church's Mechanics of Engineering. 8vo, 6 00
Du Bois's Mechanics of Engineering. Vol. I. ....Small 4to, 7 50
*Eckel's Cements, Limes, and Plasters...............................8vo, 6 00

Johnson's Materials of Construction.	Large 8vo,	6 00
Fowler's Ordinary Foundations.	8vo,	3 50
Graves's Forest Mensuration.	8vo,	4 00
* Greene's Structural Mechanics.	8vo,	2 50
Keep's Cast Iron.	8vo,	2 50
Lanza's Applied Mechanics.	8vo,	7 50
Martens's Handbook on Testing Materials. (Henning.) 2 vols.	8vo,	7 50
Maurer's Technical Mechanics.	8vo,	4 00
Merrill's Stones for Building and Decoration.	8vo,	5 00
Merriman's Mechanics of Materials.	8vo,	5 00
* Strength of Materials	12mo,	1 00
Metcalf's Steel. A Manual for Steel-users.	12mo,	2 00
Patton's Practical Treatise on Foundations.	8vo,	5 00
Richardson's Modern Asphalt Pavements.	8vo,	3 00
Richey's Handbook for Superintendents of Construction.	16mo, mor.,	4 00
* Ries's Clays: Their Occurrence, Properties, and Uses.	8vo,	5 00
Rockwell's Roads and Pavements in France.	12mo,	1 25
Sabin's Industrial and Artistic Technology of Paints and Varnish.	8vo,	3 00
* Schwarz's Longleaf Pine in Virgin Forest	12mo,	1 25
Smith's Materials of Machines.	12mo,	1 00
Snow's Principal Species of Wood.	8vo,	3 50
Spalding's Hydraulic Cement.	12mo,	2 00
Text-book on Roads and Pavements.	12mo,	2 00
Taylor and Thompson's Treatise on Concrete, Plain and Reinforced.	8vo,	5 00
Thurston's Materials of Engineering. 3 Parts.	8vo,	8 00
Part I. Non-metallic Materials of Engineering and Metallurgy.	8vo,	2 00
Part II. Iron and Steel.	8vo,	3 50
Part III. A Treatise on Brasses, Bronzes, and Other Alloys and their Constituents.	8vo,	2 50
Tillson's Street Pavements and Paving Materials.	8vo,	4 00
Turneaure and Maurer's Principles of Reinforced Concrete Construction.	8vo,	3 00
Waddell's De Pontibus. (A Pocket-book for Bridge Engineers.)	16mo, mor.,	2 00
* Specifications for Steel Bridges.	12mo,	50
Wood's (De V.) Treatise on the Resistance of Materials, and an Appendix on the Preservation of Timber.	8vo,	2 00
Wood's (De V.) Elements of Analytical Mechanics.	8vo,	3 00
Wood's (M. P.) Rustless Coatings: Corrosion and Electrolysis of Iron and Steel.	8vo,	4 00

RAILWAY ENGINEERING.

Andrew's Handbook for Street Railway Engineers.	3x5 inches, morocco,	1 25
Berg's Buildings and Structures of American Railroads	4to,	5 00
Brook's Handbook of Street Railroad Location.	16mo, morocco,	1 50
Butt's Civil Engineer's Field-book.	16mo, morocco,	2 50
Crandall's Transition Curve.	16mo, morocco,	1 50
Railway and Other Earthwork Tables.	8vo,	1 50
Crockett's Methods for Earthwork Computations. (In Press)		
Dawson's "Engineering" and Electric Traction Pocket-book	16mo, morocco	5 00
Dredge's History of the Pennsylvania Railroad: (1879)	Paper,	5 00
Fisher's Table of Cubic Yards.	Cardboard,	25
Godwin's Railroad Engineers' Field-book and Explorers' Guide.	16mo, mor.,	2 50
Hudson's Tables for Calculating the Cubic Contents of Excavations and Embankments.	8vo,	1 00
Molitor and Beard's Manual for Resident Engineers.	16mo,	1 00
Nagle's Field Manual for Railroad Engineers.	16mo, morocco,	3 00
Philbrick's Field Manual for Engineers.	16mo, morocco,	3 00
Raymond's Elements of Railroad Engineering. (In Press.)		

Searles's Field Engineering..........................16mo, morocco, 3 00
Railroad Spiral..................................16mo, morocco, 1 50
Taylor's Prismoidal Formulæ and Earthwork........................8vo, 1 50
* Trautwine's Method of Calculating the Cube Contents of Excavations and Embankments by the Aid of Diagrams........................8vo, 2 00
The Field Practice of Laying Out Circular Curves for Railroads. 12mo, morocco, 2 50
Cross-section Sheet..Paper, 25
Webb's Railroad Construction..........................16mo, morocco, 5 00
Economics of Railroad Construction....................Large 12mo, 2 50
Wellington's Economic Theory of the Location of Railways......Small 8vo, 5 00

DRAWING.

Barr's Kinematics of Machinery..8vo, 2 50
* Bartlett's Mechanical Drawing..8vo, 3 00
* " " " Abridged Ed......................8vo, 1 50
Coolidge's Manual of Drawing................................8vo, paper, 1 00
Coolidge and Freeman's Elements of General Drafting for Mechanical Engineers..Oblong 4to, 2 50
Durley's Kinematics of Machines..8vo, 4 00
Emch's Introduction to Projective Geometry and its Applications........8vo, 2 50
Hill's Text-book on Shades and Shadows, and Perspective..............8vo, 2 00
Jamison's Elements of Mechanical Drawing..............................8vo, 2 50
Advanced Mechanical Drawing..8vo, 2 00
Jones's Machine Design:
Part I. Kinematics of Machinery..............................8vo, 1 50
Part II. Form, Strength, and Proportions of Parts..............8vo, 3 00
MacCord's Elements of Descriptive Geometry............................8vo, 3 00
Kinematics; or, Practical Mechanism..............................8vo, 5 00
Mechanical Drawing..4to, 4 00
Velocity Diagrams..8vo, 1 50
MacLeod's Descriptive Geometry..................................Small 8vo, 1 50
* Mahan's Descriptive Geometry and Stone-cutting....................8vo, 1 50
Industrial Drawing. (Thompson.)..............................8vo, 3 50
Moyer's Descriptive Geometry..8vo, 2 00
Reed's Topographical Drawing and Sketching..........................4to, 5 00
Reid's Course in Mechanical Drawing..................................8vo, 2 00
Text-book of Mechanical Drawing and Elementary Machine Design. 8vo, 3 00
Robinson's Principles of Mechanism....................................8vo, 3 00
Schwamb and Merrill's Elements of Mechanism..........................8vo, 3 00
Smith's (R. S.) Manual of Topographical Drawing. (McMillan.).......8vo, 2 50
Smith (A. W.) and Marx's Machine Design..............................8vo, 3 00
* Titsworth's Elements of Mechanical Drawing..................Oblong 8vo, 1 25
Warren's Elements of Plane and Solid Free-hand Geometrical Drawing. 12mo, 1 00
Drafting Instruments and Operations..............................12mo, 1 25
Manual of Elementary Projection Drawing......................12mo, 1 50
Manual of Elementary Problems in the Linear Perspective of Form and Shadow..12mo, 1 00
Plane Problems in Elementary Geometry..........................12mo, 1 25
Elements of Descriptive Geometry, Shadows, and Perspective.......8vo, 3 50
General Problems of Shades and Shadows........................8vo, 3 00
Elements of Machine Construction and Drawing....................8vo, 7 50
Problems, Theorems, and Examples in Descriptive Geometry.......8vo, 2 50
Weisbach's Kinematics and Power of Transmission. (Hermann and Klein.)..8vo, 5 00
Whelpley's Practical Instruction in the Art of Letter Engraving.......12mo, 2 00
Wilson's (H. M.) Topographic Surveying..............................8vo, 3 50

Wilson's (V. T.) Free-hand Perspective. 8vo, 2 50
Wilson's (V. T.) Free-hand Lettering. 8vo, 1 00
Woolf's Elementary Course in Descriptive Geometry. . . . . Large 8vo, 3 00

ELECTRICITY AND PHYSICS.

* Abegg's Theory of Electrolytic Dissociation. (Von Ende.) 12mo, 1 25
Anthony and Brackett's Text-book of Physics. (Magie.) . . . . Small 8vo, 3 00
Anthony's Lecture-notes on the Theory of Electrical Measurements. . . . 12mo, 1 00
Benjamin's History of Electricity. 8vo, 3 00
Voltaic Cell. 8vo, 3 00
Betts's Lead Refining and Electrolysis. (In Press.)
Classen's Quantitative Chemical Analysis by Electrolysis. (Boltwood.) .8vo, 3 00
* Collins's Manual of Wireless Telegraphy. 12mo, 1 50
Morocco, 2 00
Crehore and Squier's Polarizing Photo-chronograph. 8vo, 3 00
* Danneel's Electrochemistry. (Merriam.) 12mo, 1 25
Dawson's "Engineering" and Electric Traction Pocket-book. 16mo, morocco, 5 00
Dolezalek's Theory of the Lead Accumulator (Storage Battery). (Von Ende.)
12mo, 2 50
Duhem's Thermodynamics and Chemistry. (Burgess.) 8vo, 4 00
Flather's Dynamometers, and the Measurement of Power. 12mo, 3 00
Gilbert's De Magnete. (Mottelay.) 8vo, 2 50
Hanchett's Alternating Currents Explained. 12mo, 1 00
Hering's Ready Reference Tables (Conversion Factors) . . . . . . 16mo, morocco, 2 50
Hobart and Ellis's High-speed Dynamo Electric Machinery. (In Press.)
Holman's Precision of Measurements. 8vo, 2 00
Telescopic Mirror-scale Method, Adjustments, and Tests. . . . Large 8vo, 75
Karapetoff's Experimental Electrical Engineering. (In Press.)
Kinzbrunner's Testing of Continuous-current Machines. 8vo, 2 00
Landauer's Spectrum Analysis. (Tingle.) 8vo, 3 00
Le Chatelier's High-temperature Measurements. (Boudouard—Burgess.) 12mo, 3 00
Löb's Electrochemistry of Organic Compounds. (Lorenz.) 8vo, 3 00
* Lyons's Treatise on Electromagnetic Phenomena. Vols. I. and II. 8vo, each, 6 00
* Michie's Elements of Wave Motion Relating to Sound and Light. 8vo, 4 00
Niaudet's Elementary Treatise on Electric Batteries. (Fishback.) 12mo, 2 50
Norris's Introduction to the Study of Electrical Engineering. (In Press.)
* Parshall and Hobart's Electric Machine Design . . . . 4to, half morocco, 12 50
Reagan's Locomotives: Simple, Compound, and Electric. New Edition.
Large 12mo, 3 50
* Rosenberg's Electrical Engineering. (Haldane Gee—Kinzbrunner.) . . .8vo, 2 00
Ryan, Norris, and Hoxie's Electrical Machinery. Vol. I. 8vo, 2 50
Thurston's Stationary Steam-engines. 8vo, 2 50
* Tillman's Elementary Lessons in Heat. 8vo, 1 50
Tory and Pitcher's Manual of Laboratory Physics. . . . . Small 8vo, 2 00
Ulke's Modern Electrolytic Copper Refining. 8vo, 3 00

LAW.

* Davis's Elements of Law. 8vo, 2 50
* Treatise on the Military Law of United States. .8vo, 7 00
* Sheep, 7 50
* Dudley's Military Law and the Procedure of Courts-martial . . . Large 12mo, 2 50
Manual for Courts-martial. . . . . 16mo, morocco, 1 50
Wait's Engineering and Architectural Jurisprudence. 8vo, 6 00
Sheep, 6 50
Law of Operations Preliminary to Construction in Engineering and Architecture. 8vo 5 00
Sheep, 5 50
Law of Contracts. 8vo, 3 00
Winthrop's Abridgment of Military Law. 12mo, 2 50

MANUFACTURES.

Bernadou's Smokeless Powder—Nitro-cellulose and Theory of the Cellulose Molecule	12mo,	2 50
Bolland's Iron Founder	12mo,	2 50
"The Iron Founder," Supplement	12mo,	[illegible] 50
Encyclopedia of Founding and Dictionary of Foundry Terms Used in the Practice of Moulding	12mo,	3 00
* Claassen's Beet-sugar Manufacture. (Hall and Rolfe.)	8vo,	3 00
* Eckel's Cements, Limes, and Plasters	8vo,	6 00
Eissler's Modern High Explosives	8vo,	4 00
Effront's Enzymes and their Applications. (Prescott.)	8vo,	3 00
Fitzgerald's Boston Machinist	12mo,	1 00
Ford's Boiler Making for Boiler Makers	18mo,	1 00
Herrick's Denatured or Industrial Alcohol	8vo,	4 00
Holley and Ladd's Analysis of Mixed Paints, Color Pigments, and Varnishes. (In Press.)		
Hopkins's Oil-chemists' Handbook	8vo,	3 00
Keep's Cast Iron	8vo,	2 50
Leach's The Inspection and Analysis of Food with Special Reference to State Control	Large 8vo,	7 50
* McKay and Larsen's Principles and Practice of Butter-making	8vo,	1 50
Maire's Modern Pigments and their Vehicles. (In Press.)		
Matthews's The Textile Fibres. 2d Edition, Rewritten	8vo,	4 00
Metcalf's Steel. A Maunal for Steel-users	12mo,	2 00
Metcalfe's Cost of Manufactures—And the Administration of Workshops	8vo,	5 00
Meyer's Modern Locomotive Construction	4to,	10 00
Morse's Calculations used in Cane-sugar Factories	16mo, morocco,	1 50
* Reisig's Guide to Piece-dyeing	8vo,	25 00
Rice's Concrete-block Manufacture	8vo,	2 00
Sabin's Industrial and Artistic Technology of Paints and Varnish	8vo,	3 00
Smith's Press-working of Metals	8vo,	3 00
Spalding's Hydraulic Cement	12mo,	2 00
Spencer's Handbook for Chemists of Beet-sugar Houses	16mo, morocco,	3 00
Handbook for Cane Sugar Manufacturers	16mo, morocco,	3 00
Taylor and Thompson's Treatise on Concrete, Plain and Reinforced	8vo,	5 00
Thurston's Manual of Steam-boilers, their Designs, Construction and Operation	8vo,	5 00
Ware's Beet-sugar Manufacture and Refining. Vol. I	Small 8vo,	4 00
" " " " " Vol. II	8vo,	5 00
Weaver's Military Explosives	8vo,	3 00
West's American Foundry Practice	12mo,	2 50
Moulder's Text-book	12mo,	2 50
Wolff's Windmill as a Prime Mover	8vo,	3 00
Wood's Rustless Coatings: Corrosion and Electrolysis of Iron and Steel	8vo,	4 00

MATHEMATICS.

Baker's Elliptic Functions	8vo,	1 50
Briggs's Elements of Plane Analytic Geometry	12mo,	1 00
Buchanan's Plane and Spherical Trigonometry. (In Press.)		
Compton's Manual of Logarithmic Computations	12mo,	1 50
Davis's Introduction to the Logic of Algebra	8vo,	1 50
* Dickson's College Algebra	Large 12mo,	1 50
* Introduction to the Theory of Algebraic Equations	Large 12mo,	1 25
Emch's Introduction to Projective Geometry and its Applications	8vo,	2 50
Halsted's Elements of Geometry	8vo,	1 75
Elementary Synthetic Geometry	8vo,	1 50
* Rational Geometry	12mo,	1 50

* Johnson's (J. B.) Three-place Logarithmic Tables: Vest-pocket size. paper,		15
	100 copies for	5 00
* Mounted on heavy cardboard, 8 × 10 inches,		25
	10 copies for	2 00
Johnson's (W. W.) Elementary Treatise on Differential Calculus	Small 8vo,	3 00
Elementary Treatise on the Integral Calculus	Small 8vo,	1 50
Johnson's (W. W.) Curve Tracing in Cartesian Co-ordinates	12mo,	1 00
Johnson's (W. W.) Treatise on Ordinary and Partial Differential Equations.	Small 8vo,	3 50
Johnson's Treatise on the Integral Calculus	Small 8vo,	3 00
Johnson's (W. W.) Theory of Errors and the Method of Least Squares	12mo,	1 50
* Johnson's (W. W.) Theoretical Mechanics	12mo,	3 00
Laplace's Philosophical Essay on Probabilities. (Truscott and Emory.)	12mo,	2 00
* Ludlow and Bass. Elements of Trigonometry and Logarithmic and Other Tables	8vo,	3 00
Trigonometry and Tables published separately	Each,	2 00
* Ludlow's Logarithmic and Trigonometric Tables	8vo,	1 00
Manning's Irrational Numbers and their Representation by Sequences and Series	12mo,	1 25
Mathematical Monographs. Edited by Mansfield Merriman and Robert S. Woodward	Octavo, each	1 00

No. 1. History of Modern Mathematics, by David Eugene Smith. No. 2. Synthetic Projective Geometry, by George Bruce Halsted. No. 3. Determinants, by Laenas Gifford Weld. No. 4. Hyperbolic Functions, by James McMahon. No. 5. Harmonic Functions, by William E. Byerly. No. 6. Grassmann's Space Analysis, by Edward W. Hyde. No. 7. Probability and Theory of Errors, by Robert S. Woodward. No. 8. Vector Analysis and Quaternions, by Alexander Macfarlane. No. 9. Differential Equations, by William Woolsey Johnson. No. 10. The Solution of Equations, by Mansfield Merriman. No. 11. Functions of a Complex Variable, by Thomas S. Fiske.

Maurer's Technical Mechanics	8vo,	4 00
Merriman's Method of Least Squares	8vo,	2 00
Rice and Johnson's Elementary Treatise on the Differential Calculus	Sm. 8vo,	3 00
Differential and Integral Calculus. 2 vols. in one	Small 8vo,	2 50
* Veblen and Lennes's Introduction to the Real Infinitesimal Analysis of One Variable	8vo,	2 00
Wood's Elements of Co-ordinate Geometry	8vo,	2 00
Trigonometry: Analytical, Plane, and Spherical	12mo,	1 00

MECHANICAL ENGINEERING.

MATERIALS OF ENGINEERING, STEAM-ENGINES AND BOILERS.

Bacon's Forge Practice	12mo,	1 50
Baldwin's Steam Heating for Buildings	12mo,	2 50
Barr's Kinematics of Machinery	8vo,	2 50
* Bartlett's Mechanical Drawing	8vo,	3 00
* " " " Abridged Ed.	8vo,	1 50
Benjamin's Wrinkles and Recipes	12mo,	2 00
Carpenter's Experimental Engineering	8vo,	6 00
Heating and Ventilating Buildings	8vo,	4 00
Clerk's Gas and Oil Engine	Small 8vo,	4 00
Coolidge's Manual of Drawing	8vo, paper,	1 00
Coolidge and Freeman's Elements of General Drafting for Mechanical Engineers	Oblong 4to,	2 50
Cromwell's Treatise on Toothed Gearing	12mo,	1 50
Treatise on Belts and Pulleys	12mo,	1 50

Durley's Kinematics of Machines. 8vo, 4 00
Flather's Dynamometers and the Measurement of Power. 12mo, 3 00
Rope Driving. 12mo, 2 00
Gill's Gas and Fuel Analysis for Engineers. 12mo, 1 25
Hall's Car Lubrication. 12mo, 1 00
Hering's Ready Reference Tables (Conversion Factors). . . . . .16mo, morocco, 2 50
Hutton's The Gas Engine. 8vo, 5 00
Jamison's Mechanical Drawing. 8vo, 2 50
Jones's Machine Design:
Part I. Kinematics of Machinery. 8vo, 1 50
Part II. Form, Strength, and Proportions of Parts. 8vo, 3 00
Kent's Mechanical Engineers' Pocket-book. . . . . .16mo, morocco, 5 00
Kerr's Power and Power Transmission. 8vo, 2 00
Leonard's Machine Shop, Tools, and Methods. 8vo, 4 00
* Lorenz's Modern Refrigerating Machinery. (Pope, Haven, and Dean.) . .8vo, 4 00
MacCord's Kinematics; or, Practical Mechanism. 8vo, 5 00
Mechanical Drawing. 4to, 4 00
Velocity Diagrams. 8vo, 1 50
MacFarland's Standard Reduction Factors for Gases. 8vo, 1 50
Mahan's Industrial Drawing. (Thompson.). 8vo, 3 50
Poole's Calorific Power of Fuels. 8vo, 3 00
Reid's Course in Mechanical Drawing. 8vo, 2 00
Text-book of Mechanical Drawing and Elementary Machine Design. 8vo, 3 00
Richard's Compressed Air 12mo, 1 50
Robinson's Principles of Mechanism. 8vo, 3 00
Schwamb and Merrill's Elements of Mechanism. 8vo, 3 00
Smith's (O.) Press-working of Metals. 8vo, 3 00
Smith (A. W.) and Marx's Machine Design. 8vo, 3 00
Thurston's Treatise on Friction and Lost Work in Machinery and Mill Work. 8vo, 3 00
Animal as a Machine and Prime Motor, and the Laws of Energetics. 12mo, 1 00
Tillson's Complete Automobile Instructor 16mo, 1 50
Morocco, 2 00
Warren's Elements of Machine Construction and Drawing. 8vo, 7 50
Weisbach's Kinematics and the Power of Transmission. (Herrmann—Klein.). 8vo, 5 00
Machinery of Transmission and Governors. (Herrmann—Klein.). .8vo, 5 00
Wolff's Windmill as a Prime Mover. 8vo, 3 00
Wood's Turbines. 8vo, 2 50

MATERIALS OF ENGINEERING.

* Bovey's Strength of Materials and Theory of Structures. 8vo, 7 50
Burr's Elasticity and Resistance of the Materials of Engineering. 6th Edition. Reset. 8vo, 7 50
Church's Mechanics of Engineering. 8vo, 6 00
* Greene's Structural Mechanics 8vo, 2 50
Johnson's Materials of Construction. 8vo, 6 00
Keep's Cast Iron. 8vo, 2 50
Lanza's Applied Mechanics. 8vo, 7 50
Martens's Handbook on Testing Materials. (Henning.) 8vo, 7 50
Maurer's Technical Mechanics. 8vo, 4 00
Merriman's Mechanics of Materials. 8vo, 5 00
* Strength of Materials 12mo, 1 00
Metcalf's Steel. A Manual for Steel-users. 12mo, 2 00
Sabin's Industrial and Artistic Technology of Paints and Varnish. 8vo, 3 00
Smith's Materials of Machines. 12mo, 1 00
Thurston's Materials of Engineering. . . . . .3 vols., 8vo, 8 00
Part II. Iron and Steel. 8vo, 3 50
Part III. A Treatise on Brasses, Bronzes, and Other Alloys and their Constituents. 8vo, 2 50

Wood's (De V.) Treatise on the Resistance of Materials and an Appendix on the Preservation of Timber. 8vo, 2 00
Elements of Analytical Mechanics. 8vo, 3 00
Wood's (M. P.) Rustless Coatings: Corrosion and Electrolysis of Iron and Steel. 8vo, 4 00

STEAM-ENGINES AND BOILERS.

Berry's Temperature-entropy Diagram. 12mo, 1 25
Carnot's Reflections on the Motive Power of Heat. (Thurston.). 12mo, 1 50
Creighton's Steam-engine and other Heat-motors 8vo, 5 00
Dawson's "Engineering" and Electric Traction Pocket-book. . . .16mo, mor., 5 00
Ford's Boiler Making for Boiler Makers. 18mo, 1 00
Goss's Locomotive Sparks. 8vo, 2 00
Locomotive Performance 8vo, 5 00
Hemenway's Indicator Practice and Steam-engine Economy. 12mo, 2 00
Hutton's Mechanical Engineering of Power Plants. 8vo, 5 00
Heat and Heat-engines. 8vo, 5 00
Kent's Steam boiler Economy. 8vo, 4 00
Kneass's Practice and Theory of the Injector. 8vo, 1 50
MacCord's Slide-valves. 8vo, 2 00
Meyer's Modern Locomotive Construction. 4to, 10 00
Peabody's Manual of the Steam-engine Indicator. 12mo, 1 50
Tables of the Properties of Saturated Steam and Other Vapors. 8vo, 1 00
Thermodynamics of the Steam-engine and Other Heat-engines. 8vo, 5 00
Valve-gears for Steam-engines. 8vo, 2 50
Peabody and Miller's Steam-boilers. 8vo, 4 00
Pray's Twenty Years with the Indicator. . . . . .Large 8vo, 2 50
Pupin's Thermodynamics of Reversible Cycles in Gases and Saturated Vapors. (Osterberg.). 12mo, 1 25
Reagan's Locomotives: Simple, Compound, and Electric. New Edition. Large 12mo, 3 50
Sinclair's Locomotive Engine Running and Management. 12mo, 2 00
Smart's Handbook of Engineering Laboratory Practice. 12mo, 2 50
Snow's Steam-boiler Practice. 8vo, 3 00
Spangler's Valve-gears. 8vo, 2 50
Notes on Thermodynamics. 12mo, 1 00
Spangler, Greene, and Marshall's Elements of Steam-engineering 8vo, 3 00
Thomas's Steam-turbines. 8vo, 3 50
Thurston's Handy Tables. 8vo, 1 50
Manual of the Steam-engine. . . . . .2 vols., 8vo, 10 00
Part I. History, Structure, and Theory. 8vo, 6 00
Part II. Design, Construction, and Operation. 8vo, 6 00
Handbook of Engine and Boiler Trials, and the Use of the Indicator and the Prony Brake. 8vo, 5 00
Stationary Steam-engines. 8vo, 2 50
Steam-boiler Explosions in Theory and in Practice 12mo, 1 50
Manual of Steam-boilers, their Designs, Construction, and Operation .8vo, 5 00
Wehrenfenning's Analysis and Softening of Boiler Feed-water (Patterson) 8vo, 4 00
Weisbach's Heat, Steam, and Steam-engines. (Du Bois.). 8vo, 5 00
Whitham's Steam-engine Design. 8vo, 5 00
Wood's Thermodynamics, Heat Motors, and Refrigerating Machines. . .8vo, 4 00

MECHANICS AND MACHINERY.

Barr's Kinematics of Machinery. 8vo, 2 50
* Bovey's Strength of Materials and Theory of Structures 8vo, 7 50
Chase's The Art of Pattern-making. 12mo, 2 50

Church's Mechanics of Engineering. 8vo, 6 00
Notes and Examples in Mechanics. 8vo, 2 00
Compton's First Lessons in Metal-working. 12mo, 1 50
Compton and De Groodt's The Speed Lathe 12mo, 1 50
Cromwell's Treatise on Toothed Gearing. 12mo, 1 50
Treatise on Belts and Pulleys. 12mo, 1 50
Dana's Text-book of Elementary Mechanics for Colleges and Schools. .12mo, 1 50
Dingey's Machinery Pattern Making 12mo, 2 00
Dredge's Record of the Transportation Exhibits Building of the World's Columbian Exposition of 1893. . . . . .4to half morocco, 5 00
Du Bois's Elementary Principles of Mechanics:
Vol. I. Kinematics. 8vo, 3 50
Vol. II. Statics. 8vo, 4 00
Mechanics of Engineering. Vol. I. . . . . .Small 4to, 7 50
Vol. II. . . . . .Small 4to, 10 00
Durley's Kinematics of Machines. 8vo, 4 00
Fitzgerald's Boston Machinist. 16mo, 1 00
Flather's Dynamometers, and the Measurement of Power. 12mo, 3 00
Rope Driving. 12mo, 2 00
Goss's Locomotive Sparks. 8vo, 2 00
Locomotive Performance. 8vo, 5 00
* Greene's Structural Mechanics. 8vo, 2 50
Hall's Car Lubrication. 12mo, 1 00
Hobart and Ellis's High-speed Dynamo Electric Machinery. (In Press.)
Holly's Art of Saw Filing. 18mo, 75
James's Kinematics of a Point and the Rational Mechanics of a Particle. Small 8vo, 2 00
* Johnson's (W. W.) Theoretical Mechanics. 12mo, 3 00
Johnson's (L. J.) Statics by Graphic and Algebraic Methods. 8vo, 2 00
Jones's Machine Design:
Part I. Kinematics of Machinery. 8vo, 1 50
Part II. Form, Strength, and Proportions of Parts. 8vo, 3 00
Kerr's Power and Power Transmission. 8vo, 2 00
Lanza's Applied Mechanics. 8vo, 7 50
Leonard's Machine Shop, Tools, and Methods. 8vo, 4 00
* Lorenz's Modern Refrigerating Machinery. (Pope, Haven, and Dean.).8vo, 4 00
MacCord's Kinematics; or, Practical Mechanism. 8vo, 5 00
Velocity Diagrams. 8vo, 1 50
* Martin's Text Book on Mechanics, Vol. I, Statics. 12mo, 1 25
* Vol. 2, Kinematics and Kinetics . .12mo, 1 50
Maurer's Technical Mechanics. 8vo, 4 00
Merriman's Mechanics of Materials. 8vo, 5 00
* Elements of Mechanics. 12mo, 1 00
* Michie's Elements of Analytical Mechanics. 8vo, 4 00
* Parshall and Hobart's Electric Machine Design. . . . . .4to, half morocco, 12 50
Reagan's Locomotives: Simple, Compound, and Electric. New Edition. Large 12mo, 3 50
Reid's Course in Mechanical Drawing. 8vo, 2 00
Text-book of Mechanical Drawing and Elementary Machine Design.8vo, 3 00
Richards's Compressed Air. 12mo, 1 50
Robinson's Principles of Mechanism. 8vo, 3 00
Ryan, Norris, and Hoxie's Electrical Machinery. Vol. I. 8vo, 2 50
Sanborn's Mechanics: Problems. . . . . .Large 12mo, 1 50
Schwamb and Merrill's Elements of Mechanism. 8vo, 3 00
Sinclair's Locomotive-engine Running and Management. 12mo, 2 00
Smith's (O.) Press-working of Metals 8vo, 3 00
Smith's (A. W.) Materials of Machines. 12mo, 1 00
Smith (A. W.) and Marx's Machine Design. 8vo, 3 00
Sorel's Carbureting and Combustion of Alcohol Engines. (Woodward and Preston.). . . . . .Large 8vo, 3 00

Spangler, Greene, and Marshall's Elements of Steam-engineering	8vo,	3 00
Thurston's Treatise on Friction and Lost Work in Machinery and Mill Work	8vo,	3 00
Animal as a Machine and Prime Motor, and the Laws of Energetics.	12mo,	1 00
Tillson's Complete Automobile Instructor	16mo,	1 50
	Morocco,	2 00
Warren's Elements of Machine Construction and Drawing	8vo,	7 50
Weisbach's Kinematics and Power of Transmission. (Herrmann—Klein.)	8vo.	5 00
Machinery of Transmission and Governors. (Herrmann—Klein.)	8vo.	5 00
Wood's Elements of Analytical Mechanics.	8vo,	3 00
Principles of Elementary Mechanics	12mo,	1 25
Turbines	8vo,	2 50
The World's Columbian Exposition of 1893	4to,	1 00

MEDICAL.

* Bolduan's Immune Sera	12mo,	1 50
De Fursac's Manual of Psychiatry. (Rosanoff and Collins.)	Large 12mo,	2 50
Ehrlich's Collected Studies on Immunity. (Bolduan.)	8vo,	6 00
* Fischer's Physiology of Alimentation	Large 12mo, cloth,	2 00
Hammarsten's Text-book on Physiological Chemistry. (Mandel.)	8vo,	4 00
Lassar-Cohn's Practical Urinary Analysis. (Lorenz.)	12mo,	1 00
* Pauli's Physical Chemistry in the Service of Medicine. (Fischer.)	12mo,	1 25
* Pozzi-Escot's The Toxins and Venoms and their Antibodies. (Cohn.)	12mo,	1 00
Rostoski's Serum Diagnosis. (Bolduan.)	12mo,	1 00
Salkowski's Physiological and Pathological Chemistry. (Orndorff.)	8vo,	2 50
* Satterlee's Outlines of Human Embryology	12mo,	1 25
Steel's Treatise on the Diseases of the Dog.	8vo,	3 50
Von Behring's Suppression of Tuberculosis. (Bolduan.)	12mo,	1 00
Woodhull's Notes on Military Hygiene	16mo,	1 50
* Personal Hygiene	12mo,	1 00
Wulling's An Elementary Course in Inorganic Pharmaceutical and Medical Chemistry	12mo,	2 00

METALLURGY.

Betts's Lead Refining by Electrolysis. (In Press.)		
Egleston's Metallurgy of Silver, Gold, and Mercury:		
Vol. I. Silver	8vo,	7 50
Vol. II. Gold and Mercury	8vo,	7 50
Goesel's Minerals and Metals: A Reference Book	16mo, mor.	3 00
* Iles's Lead-smelting	12mo,	2 50
Keep's Cast Iron	8vo,	2 50
Kunhardt's Practice of Ore Dressing in Europe	8vo,	1 50
Le Chatelier's High-temperature Measurements. (Boudouard—Burgess.)	12mo,	3 00
Metcalf's Steel. A Manual for Steel-users	12mo,	2 00
Miller's Cyanide Process	12mo,	1 00
Minet's Production of Aluminum and its Industrial Use. (Waldo.)	12mo,	2 50
Robine and Lenglen's Cyanide Industry. (Le Clerc.)	8vo,	4 00
Smith's Materials of Machines.	12mo,	1 00
Thurston's Materials of Engineering. In Three Parts	8vo,	8 00
Part II. Iron and Steel	8vo,	3 50
Part III. A Treatise on Brasses, Bronzes, and Other Alloys and their Constituents	8vo,	2 50
Ulke's Modern Electrolytic Copper Refining	8vo,	3 00

MINERALOGY.

Barringer's Description of Minerals of Commercial Value.	Oblong, morocco,	2 50
Boyd's Resources of Southwest Virginia	8vo,	3 00

Boyd's Map of Southwest Virignia........................Pocket-book form. 2 00
* Browning's Introduction to the Rarer Elements..... . ... 8vo, 1 50
Brush's Manual of Determinative Mineralogy. (Penfield.).............8vo, 4 00
Chester's Catalogue of Minerals.............................8vo, paper, 1 00
Cloth, 1 25
Dictionary of the Names of Minerals...........................8vo, 3 50
Dana's System of Mineralogy.....................Large 8vo, half leather, 12 50
First Appendix to Dana's New "System of Mineralogy."......Large 8vo, 1 00
Text-book of Mineralogy.......................................8vo, 4 00
Minerals and How to Study Them..............................12mo, 1 50
Catalogue of American Localities of Minerals...............Large 8vo, 1 00
Manual of Mineralogy and Petrography........................12mo 2 00
Douglas's Untechnical Addresses on Technical Subjects..............12mo, 1 00
Eakle's Mineral Tables...8vo, 1 25
Egleston's Catalogue of Minerals and Synonyms.....................8vo, 2 50
Goesel's Minerals and Metals: A Reference Book.............16mo, mor. 3 00
Groth's Introduction to Chemical Crystallography (Marshall)........ 12mo, 1 25
Iddings's Rock Minerals..8vo, 5 00
Johannsen's Key for the Determination of Rock-forming Minerals in Thin Sections. (In Press.)
* Martin's Laboratory Guide to Qualitative Analysis with the Blowpipe.12mo, 60
Merrill's Non-metallic Minerals. Their Occurrence and Uses..........8vo, 4 00
Stones for Building and Decoration 8vo, 5 00
* Penfield's Notes on Determinative Mineralogy and Record of Mineral Tests.
8vo, paper, 50
Tables of Minerals..8vo, 1 00
* Richards's Synopsis of Mineral Characters...............12mo, morocco, 1 25
* Ries's Clays: Their Occurrence, Properties, and Uses...............8vo, 5 00
Rosenbusch's Microscopical Physiography of the Rock-making Minerals. (Iddings.)..8vo, 5 00
* Tillman's Text-book of Important Minerals and Rocks...............8vo, 2 00

MINING.

Beard's Mine Gases and Explosions. (In Press.)
Boyd's Resources of Southwest Virginia............................8vo, 3 00
Map of Southwest Virginia......................Pocket-book form, 2 00
Douglas's Untechnical Addresses on Technical Subjects..............12mo, 1 00
Eissler's Modern High Explosives...................................8vo, 4 00
Goesel's Minerals and Metals: A Reference Book.............16mo, mor. 3 00
Goodyear's Coal-mines of the Western Coast of the United States......12mo, 2 50
Ihlseng's Manual of Mining...8vo, 5 00
* Iles's Lead-smelting..12mo, 2 50
Kunhardt's Practice of Ore Dressing in Europe........................8vo, 1 50
Miller's Cyanide Process...12mo, 1 00
O'Driscoll's Notes on the Treatment of Gold Ores.....................8vo, 2 00
Robine and Lenglen's Cyanide Industry. (Le Clerc.)..................8vo, 4 00
Weaver's Military Explosives..8vo, 3 00
Wilson's Cyanide Processes...12mo, 1 50
Chlorination Process...12mo, 1 50
Hydraulic and Placer Mining. 2d edition, rewritten............12mo, 2 50
Treatise on Practical and Theoretical Mine Ventilation...........12mo, 1 25

SANITARY SCIENCE.

Bashore's Sanitation of a Country House............................12mo, 1 00
* Outlines of Practical Sanitation..... ....12mo, 1 25
Folwell's Sewerage. (Designing, Construction, and Maintenance.)......8vo, 3 00
Water-supply Engineering.......................... ....8vo, 4 00

Fowler's Sewage Works Analyses	12mo,	2 00
Fuertes's Water and Public Health	12mo,	1 50
Water-filtration Works	12mo,	2 50
Gerhard's Guide to Sanitary House-inspection	16mo,	1 00
Sanitation of Public Buildings	12mo,	1 50
Hazen's Filtration of Public Water-supplies	8vo,	3 00
Leach's The Inspection and Analysis of Food with Special Reference to State Control	8vo,	7 50
Mason's Water-supply. (Considered principally from a Sanitary Standpoint)	8vo,	4 00
Examination of Water. (Chemical and Bacteriological.)	12mo,	1 25
* Merriman's Elements of Sanitary Engineering	8vo,	2 00
Ogden's Sewer Design	12mo,	2 00
Prescott and Winslow's Elements of Water Bacteriology, with Special Reference to Sanitary Water Analysis	12mo,	1 25
* Price's Handbook on Sanitation	12mo,	1 50
Richards's Cost of Food. A Study in Dietaries	12mo,	1 00
Cost of Living as Modified by Sanitary Science	12mo,	1 00
Cost of Shelter	12mo,	1 00
Richards and Woodman's Air. Water, and Food from a Sanitary Standpoint	8vo,	2 00
* Richards and Williams's The Dietary Computer	8vo,	1 50
Rideal's Sewage and Bacterial Purification of Sewage	8vo,	4 00
Disinfection and the Preservation of Food	8vo,	4 00
Turneaure and Russell's Public Water-supplies	8vo,	5 00
Von Behring's Suppression of Tuberculosis. (Bolduan.)	12mo,	1 00
Whipple's Microscopy of Drinking-water	8vo,	3 50
Wilson's Air Conditioning. (In Press.)		
Winton's Microscopy of Vegetable Foods	8vo,	7 50
Woodhull's Notes on Military Hygiene	16mo,	1 50
* Personal Hygiene	12mo,	1 00

MISCELLANEOUS.

Association of State and National Food and Dairy Departments (Interstate Pure Food Commission):		
Tenth Annual Convention Held at Hartford, July 17–20, 1906	8vo,	3 00
Eleventh Annual Convention, Held at Jamestown Tri-Centennial Exposition, July 16–19, 1907. (In Press.)		
Emmons's Geological Guide-book of the Rocky Mountain Excursion of the International Congress of Geologists	Large 8vo,	1 50
Ferrel's Popular Treatise on the Winds	8vo,	4 00
Gannett's Statistical Abstract of the World	24mo,	75
Gerhard's The Modern Bath and Bath-houses. (In Press.)		
Haines's American Railway Management	12mo,	2 50
Ricketts's History of Rensselaer Polytechnic Institute, 1824–1894	Small 8vo,	3 00
Rotherham's Emphasized New Testament	Large 8vo,	2 00
Standage's Decorative Treatment of Wood, Glass, Metal, etc. (In Press.)		
The World's Columbian Exposition of 1893	4to,	1 00
Winslow's Elements of Applied Microscopy	12mo,	1 50

HEBREW AND CHALDEE TEXT-BOOKS.

Green's Elementary Hebrew Grammar	12mo,	1 25
Hebrew Chrestomathy	8vo,	2 00
Gesenius's Hebrew and Chaldee Lexicon to the Old Testament Scriptures. (Tregelles.)	Small 4to, half morocco,	5 00
Letteris's Hebrew Bible	8vo,	2 25